# Siddhartha Institution of Engineering & Technology

Approved by AICTE, Permanently Affiliated to JNTUH.
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## Department of Mechanical Engineering

Instructional material on

**KINEMATICS OF MACHINERY**

II B Tech – I Semester

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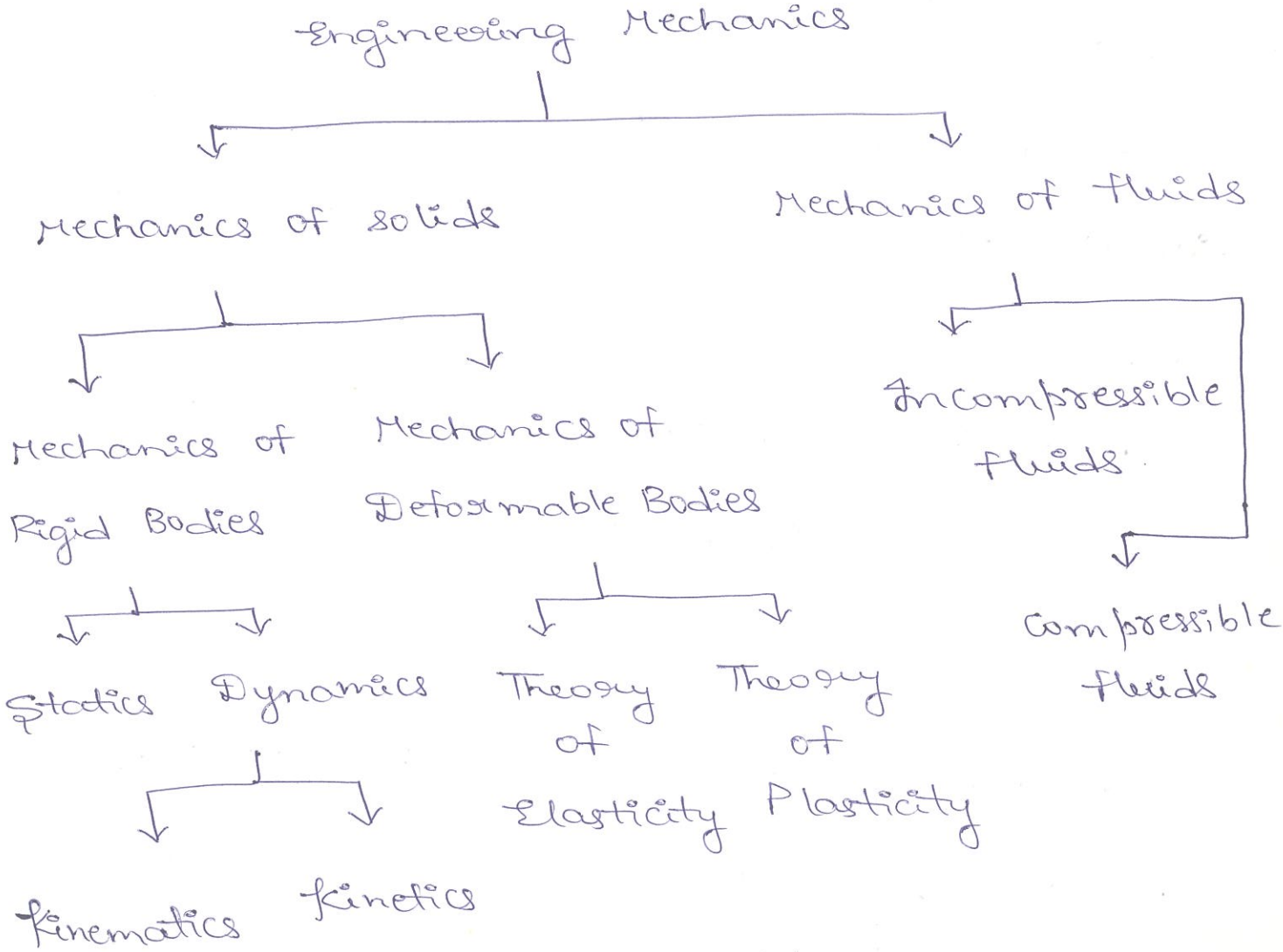
# Kinematics of Machinery.

## * Introduction :-

→ The branch of physical science that deal with the state of rest or the state of motion is termed as "Mechanics".

→ Application of laws of mechanics to field problem is termed as "Engineering Mechanics".

→ classification of EM (Block Diagram) :-



→ Rigid Bodies :-

A body is said to be rigid, if the relative positions of any two particles do not change under the action of forces.

→ Deformed Bodies :-

Bodies which deforms or change in the shape of the body are known as Deformed bodies.

→ Statics :-

The mechanics of the rigid bodies dealing with the bodies at rest is termed as statics.

→ Dynamics :-

The mechanics of the rigid bodies dealing with the bodies in motion is termed as Dynamics.

→ Kinetics :-

The dynamics dealing with the problems with referring to forces causing the motion also is called Kinetics.

→ Kinematics :-

The dynamics dealing with the problems without refering to the forces causing the motion of the body is termed as Kinematics.

We will consider displacement, velocity & acceleration quantities i.e linear motion & circular motion (Angular motion).

→ Displacement :-

The change in the position of a particle during its motion is called displacement (S).

→ velocity :-

The rate of change of displacement is called velocity (v).

→ Acceleration :-

The rate of change of velocity is called acceleration (a).

→

| linear motion | circular motion |
| --- | --- |
| $v = u + at$ | $\omega = \omega_0 + \alpha t$ |
| $S = ut + \frac{1}{2}at^2$ | $\theta = \omega_0 t + \frac{1}{2}\alpha t^2$ |
| $v^2 - u^2 = 2aS$ | $\omega^2 - \omega_0^2 = 2\alpha\theta$ |
| $S_n = u + \frac{a}{2}(2n-1)$ | $\theta_n = \omega_0 t + \frac{\alpha}{2}(2n-1)$ |

→ $V = \frac{ds}{dt}$ , $a = \frac{dv}{dt}$

→ $V = \frac{s}{t} \Rightarrow s = V_{av} t$

$\Rightarrow s = \left(\frac{V+u}{2}\right) t$

→ $\omega = \frac{d\theta}{dt}$ , $\alpha = \frac{d\omega}{dt}$

→ $\omega = \frac{\theta}{t} \Rightarrow \theta = \omega_{av} t$

$\Rightarrow \theta = \frac{(\omega + \omega_0)}{2} t$

→ $\omega = \frac{2\pi N}{60}$ rad/sec , $\frac{\theta}{2\pi} = n$

→ $s = r\theta$ , $V = r\omega$ , $a_t = r\alpha$ , $a_n = \frac{V^2}{r}$ , $a = \sqrt{a_t^2 + a_n^2}$

* A wheel accelerates uniformly from rest to 2000rpm. in 20 seconds. What is its angular acceleration & how many revolutions does the wheel make in attaining the speed of 2000rpm &

→ $N_0 = 0 \Rightarrow \omega_0 = 0$ ; $N = 2000$rpm , $t = 20$sec.

$\omega = \frac{2\pi N}{60} = \frac{2 \times \pi \times 2000}{60} = 209.5$ r/sec.

$\Rightarrow \omega = \omega_0 + \alpha t$

$\Rightarrow 209.5 = 0 + \alpha \times 20$

$$\therefore \alpha = \frac{209.5}{20} = 10.475 \text{ rad/sec}^2 \checkmark$$

$$\Rightarrow \theta = \omega_{av} t = \left(\frac{\omega_0 + \omega}{2}\right) t \quad \left(\because \omega = \frac{\theta}{t}\right).$$

$$= \left(\frac{0 + 209.5}{2}\right) 20 = 2095 \text{ rad}.$$

$$\Rightarrow n = \frac{\theta}{2\pi} = \frac{2095}{2\pi} = 333.4 \checkmark$$

* A horizontal bar 1.5 m long & of small cross-section rotates about vertical axis through one end. It accelerates uniformly from 1200 rpm. to 1500 rpm in an interval of 5 sec. What is the linear velocity at the beginning & end of the interval? What are the normal & tangential components of the acceleration of the mid-point of the bar after 5 sec after acceleration begins?

→ $l = 1.5\text{m}$, $N_0 = 1200 \text{ rpm}$, $\omega_0 = \frac{2\pi \times 1200}{60}$

$= 125.7 \text{ rad/s}$

$N = 1500 \text{ rpm}$, $\omega = \frac{2\pi \times 1500}{60}$

$= 157 \text{ rad/sec}$

$t = 5 \text{ sec}$.

$\Rightarrow V_0 = r\omega_0 = 1.5 \times 125.7 = 188.6 \text{ m/sec}$ ✓

$\Rightarrow V_5 = V = r\omega = 1.5 \times 157 = 235.5 \text{ m/sec}$ ✓

$\Rightarrow \omega = \omega_0 + \alpha t$

$157 = 125.7 + \alpha \times 5$

$\therefore \alpha = 6.26 \text{ rad/s}^2$

$\Rightarrow r_{mid} = \frac{r}{2} = \frac{1.5}{2} = 0.75 \text{ m}$

$\Rightarrow a_t = \alpha r = 6.26 \times 0.75 = 4.7 \text{ m/s}^2$ ✓

$\Rightarrow a_n / a_r = \frac{v^2}{r} = \frac{\omega^2 r^2}{r} = \omega^2 r$

$= (157)^2 \times 0.75$

$\therefore a_n / a_r = 18487 \text{ m/s}^2$ ✓

* A car starts from rest & accelerates uniformly to a speed of 72 kmph over a distance of 500m. calculate acceleration and time taken to attain the speed.

If a further acceleration raises the speed to 90 kmph in 10 sec, find this acceleration & further distance moved. The brakes are now applied to bring car to rest under uniform

retardation in 5 sec. Find distance travelled during braking.

→ 1). $u = 0$, $v = 72 \text{ kmph} = \frac{72 \times 1000}{3600} = 20 \text{ m/sec}$,

$s = 500\text{m}$, $a = ?$, $t = ?$

$\Rightarrow v^2 - u^2 = 2as$

$(20)^2 - 0 = 2a \times 500 = 1000a$

$\therefore a = \frac{(20)^2}{1000} = 0.4 \text{ m/s}^2$ ✓

$\Rightarrow v = u + at$

$20 = 0 + 0.4 \times t$

$\therefore t = \frac{20}{0.4} = 50 \text{ sec}$ ✓

2). $u = 72 \text{ km/hr} = \frac{72 \times 1000}{3600} = 20 \text{ m/sec}$,

$v = 90 \text{ km/hr} = \frac{90 \times 1000}{3600} = 25 \text{ m/sec}$,

$t = 10 \text{ sec}$, $a = ?$, $s = ?$

$\Rightarrow v = u + at$

$25 = 20 + a(10)$ $\therefore a = 0.5 \text{ m/s}^2$ ✓

$\Rightarrow s = ut + \frac{1}{2}at^2 = 20 \times 10 + \frac{1}{2} \times 0.5 (10)^2$

$\therefore s = 225\text{m}$ ✓

3). $u = 25 m/sec$, $V = 0$, $t = 5 sec$, $S = ?$

$$\Rightarrow V_{avg} = \frac{S}{t}$$

$$\Rightarrow S = \left(\frac{u+V}{2}\right) t$$

$$= \left(\frac{25+0}{2}\right) 5$$

$$\therefore S = 62.5 m.$$

* Simple Mechanisms :- (UNIT-I)

* Machine :-

Machine is a device which receives energy & transforms it into some useful work.

* Kinematic Link or Element :-

Each part of a m/c, which moves relative to some other part is known as kinematic link or element.

* Types of links :-

1). Rigid link :- No deformation (Eg:- Reciprocating steam engine)

2). Flexible link :- Partly deformation (Eg:- Belts, ropes)

3). Fluid link :- Motion is through fluid (Eg:- jacks, Hydraulic presses & brakes).

* Structure :-

It is an assemblage of no. of resistant bodies having no relative motion b/w them & meant for carrying loads having straining action. Eg:- Railway bridge, roof truss.

* Difference b/w M/c & structure :-

1). The parts of a m/c move relative to one another, whereas members of a structure do not move relative to one another.

2). A m/c transforms available energy into some useful work, whereas in a structure no energy is transformed into useful work.

3). The links of a m/c may transmit both power & motion, while members of a structure transmit forces only.

* Kinematic pair :-

→ The two links or elements of a m/c, when in contact with each other, are said to form a pair.

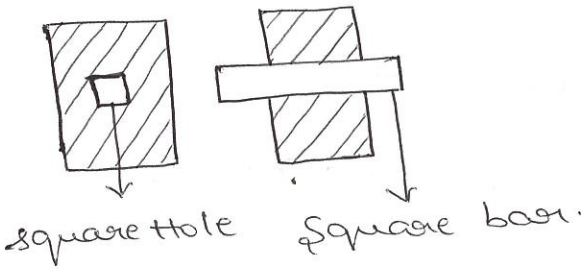
→ If relative motion b/w them is completely or successfully constrained, the pair is known

as kinematic pair.

→ Types of constrained Motions :-

1). completely constrained motion :-

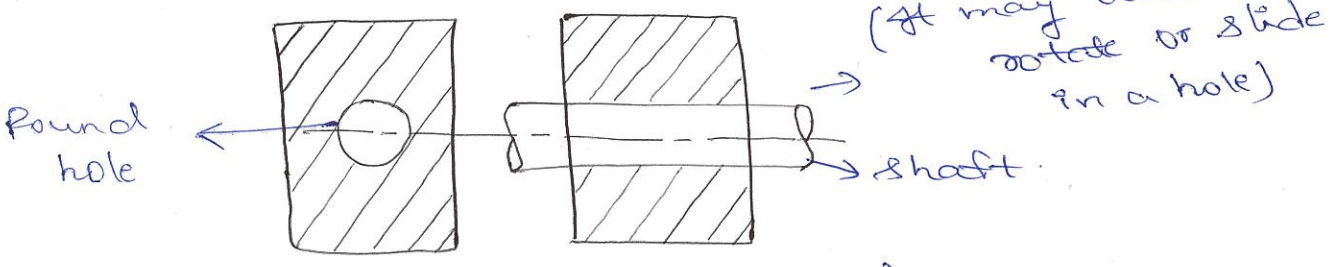
When the motion b/w a pair is limited to a definite direction irrespective of direction of force applied, then motion is said to be CSM.




(Motion of square bar in a square hole).

2). Incompletely constrained motion :-

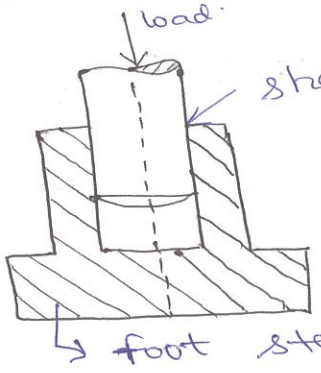
When the motion b/w a pair can take place in more than one direction, then the motion is called an incompletely constrained motion.




(Shaft in a circular hole)

3). Successfully constrained motion :-

When motion b/w elements, forming a pair, is such that constrained motion is not completed by itself, but by some other means, then motion is said to be successfully CM.




(Shaft in a foot step bearing).

* Classification of Kinematic Pairs :-

1). According to type of relative motion b/w element

→ (a). Sliding pair

(b). Turning pair

(c). Rolling pair

(d). Screw pair

(e). Spherical pair

2). According to type of contact b/w elements:-

→ (a). Lower pair

(b). Higher pair

3). According to type of closure :-

→ (a). Self closed pair

(b). Force - closed pair

* Kinematic chain :-

→ When the kinematic pairs are coupled in such a way that last link is joined to first link to transmit definite motion, it is called kinematic chain. (OR)

→ Defined as combination of kinematic pairs, joined in such a way that each link forms a part of two pairs & relative motion blw links or elements is completely or successfully constrained.

→ Each link will form two pairs. ($\because l = p$).

→ Relation b/w no. of pairs ($p$) & links ($l$) is —

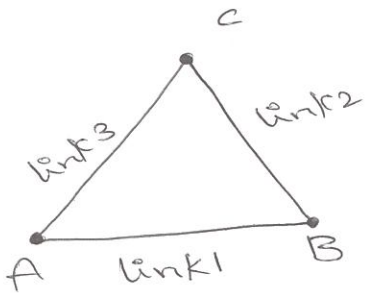
$$\Rightarrow l = 2p - 4 \longrightarrow (1).$$

→ Relation blw no. of links ($l$) & no. of joints ($j$) is —

$$\Rightarrow j = \frac{3}{2}l - 2 \longrightarrow (2).$$

→ Use these 2 eqns to determine whether chain is kinematic chain or not.

*




$l = 3$

$p = 3$

$j = 3$

→ ① ⇒ $l = 2p - 4$

$3 = 2 \times 3 - 4 = 2$

$\therefore L.H.S > R.H.S.$

② ⇒ $j = \frac{3}{2} l - 2$

$3 = \frac{3}{2} \times 3 - 2 = 2.5$

$\therefore L.H.S > R.H.S.$

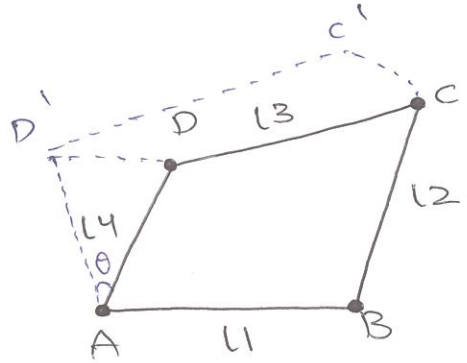
This arrangement is not satisfying 2 eqns.

so it is not a kinematic chain & no relativ

motion is possible. (Structure).

This type of chain is called "locked chain".

*




$l = 4$

$p = 4$

$j = 4$

→ ① ⇒ $l = 2p - 4$

$4 = 2 \times 4 - 4 = 4$

$\therefore L.H.S = R.H.S.$

② ⇒ $j = \frac{3}{2}l - 2$

$4 = \frac{3}{2} \times 4 - 2 = 4$

$\therefore$ L.H.S = R.H.S.

This arrangement is satisfying 2 eqn's.

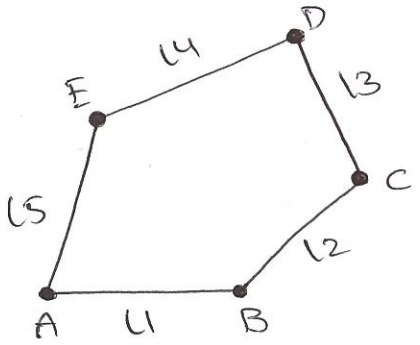
So it is a kinematic chain.

By giving definite displacement ($\theta$) to AB & keeping AB as fixed, we can get remaining displacements of BC & CD perfectly definite.

In this relative motion is completely constrained.

This type is known as "Constrained kinematic chain".

*




$l = 5$

$p = 5$

$j = 5$

→ ① ⇒ $l = 2p - 4$

$5 = 2 \times 5 - 4 = 6$.

$\therefore$ L.H.S < R.H.S

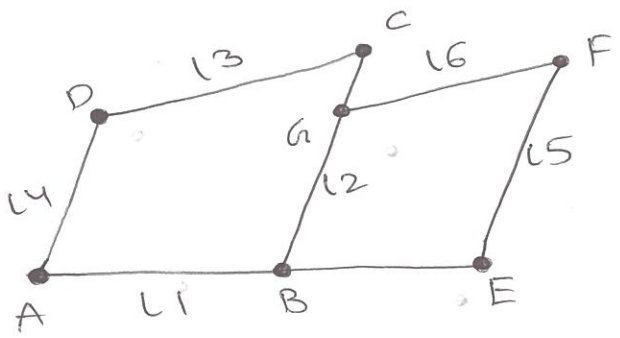
② ⇒ $j = \frac{3}{2}l - 2$

$5 = \frac{3}{2} \times 5 - 2 = 5.5$

$\therefore$ L.H.S < R.H.S.

This arrangement is not satisfying 2 eqⁿs. So it is not a kinematic chain & relative motion is not completely constrained. This type of chain is called "unconstrained chain".

*




$l = 6$

$p = 5$

$j = 7$

→ ① ⇒ $l = 2p - 4$

$6 = 2 \times 5 - 4 = 6$.

∴ L.H.S = R.H.S.

② ⇒ $j = \frac{3}{2} l - 2$

$7 = \frac{3}{2} \times 6 - 2 = 7$

∴ L.H.S = R.H.S.

This arrangement is satisfying 2 eqⁿs.

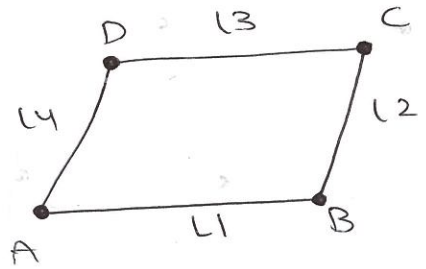
So it is a kinematic chain.

Chain having more than 4 links is known as "compound kinematic chain".

## * Types of joints in a chain :-

### * 1). Binary joint :-

If two links are joined at same connection, that joint is called binary joint.




$l = 4, \quad j = 4$

$\Rightarrow j = \frac{3}{2}l - 2$

$4 = \frac{3}{2} \times 4 - 2 = 4.$

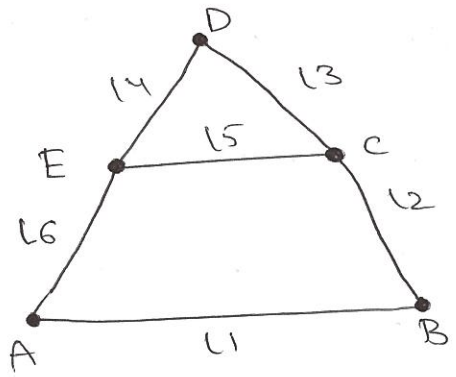
$\therefore$ L.H.S = R.H.S

$\therefore$ This is a kinematic (or) constrained chain.

& $j = 4$

### * 2). Ternary joint :-

If three links are joined at same connection, that joint is called ternary joint (1TJ = 2BJ).




$l = 6, \quad j = 7$

$\left\{ \begin{array}{l} \because \text{BJ at A, B, D} = 3 \\ \text{TJ at C, E} = 2 \end{array} \right\}$

$\Rightarrow j = 3 + 2(2)$

$3 + 4 = 7.$

$\Rightarrow j = \frac{3}{2}l - 2$

$7 = \frac{3}{2} \times 6 - 2 = 7.$

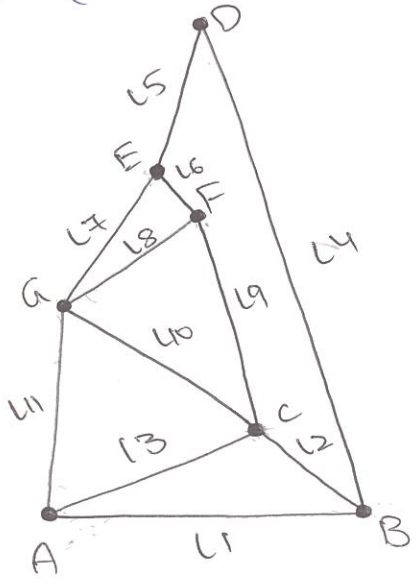
$\therefore L.H.S = R.H.S.$

$\therefore$ This is a kinematic (or) constrained chain.

& $j = 7.$

## * 3). Quaternary joint :-

If four links are joined at same connection, that joint is called quaternary joint ( 1QJ = 3BJ ).




$l = 11$

$j = 15$

$\because$ BJ at D = 1

TJ at A, B, E, F = 4

QJ at C, G = 2

$\Rightarrow j = 1 + 2(4) + 3(2)$

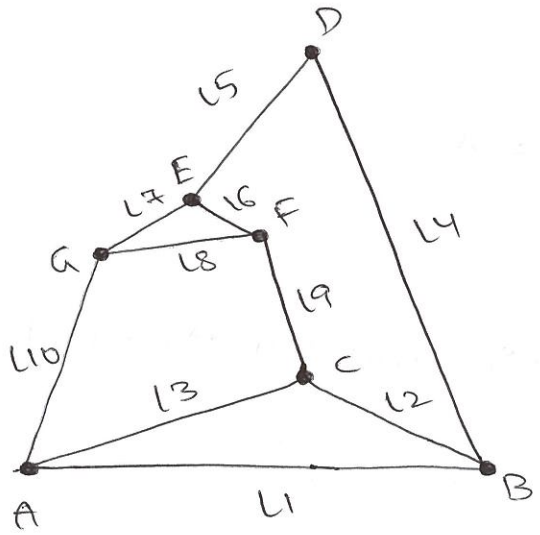
$= 15$

$\Rightarrow j = \frac{3}{2}l - 2$

$15 = \frac{3}{2} \times 11 - 2 = 14.5$

$\therefore L.H.S > R.H.S.$

$\therefore$ This is a locked chain.




Suppose link CG is removed, then -

$l = 10$

$j = 13$

{ ∵ BJ at D = 1

TJ at A, B, C, E, F, G = 6 }

$\Rightarrow j = 1 + 2(6) = 13$

$\Rightarrow j = \frac{3}{2}l - 2$

$13 = \frac{3}{2} \times 10 - 2 = 13$

∴ L.H.S = R.H.S.

∴ This is a kinematic (or) constrained chain.

* Mechanism :-

→ When one of the links of a kinematic chain is fixed, the chain is known as "mechanism".

→ A mechanism with four links is called "simple mechanism".

→ A mechanism with more than four links is called "compound mechanism".

→ When a mechanism is required to transmit power or to do some particular type of work, it then becomes a machine.

## * No. of Degrees of freedom for plane Mechanism

Max. possibility's ~~for~~ of movements (DOF = movability)

→ It is defined as no. of input parameters (variables) which must be independently controlled in order to bring mechanism into useful engg. purpose.

→ DOF can be directly determined from no. of links & no. of types of joints.

→ No. of DOF of a mechanism is given by —

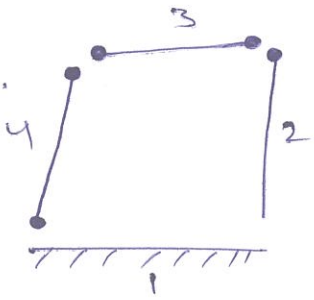
$$\Rightarrow n = 3(l-1) - 2j - h$$

(h = no. of higher pairs i.e motion b/w links is partly turning & partly sliding).

→ This eqn is called "Kutzbach criterion".

→ If h=0, then $n = 3(l-1) - 2j$.

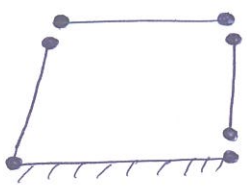
Eg:- 1).




$\Rightarrow n = 3(4-1) - 2(0)$

$\therefore n = 9.$

2).



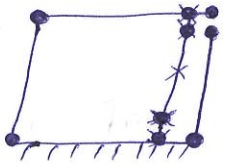
$\Rightarrow n = 3(4-1) - 2(1)$

$\therefore n = 7$

3). 

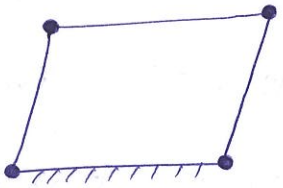
$\Rightarrow n = 3(4-1) - 2(2)$

$\therefore n = 5$

4). 

$\Rightarrow n = 3(4-1) - 2(3)$

$\therefore n = 3$

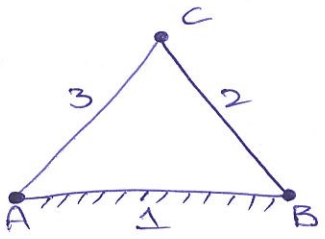
5). 

$\Rightarrow n = 3(4-1) - 2(4)$

$\therefore n = 1$

* Application of kutzbach criterion to plane Mechanisms :-

$\Rightarrow n = 3(l-1) - 2j$

1). 

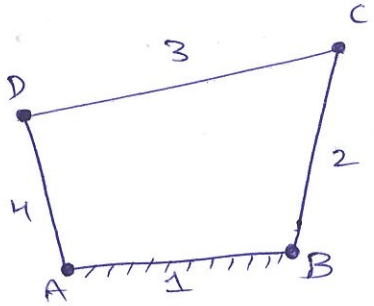

Three-bar Mechanism.

$L = 3$

$j = 3$ BJ

$\Rightarrow n = 3(3-1) - 2(3)$

$\therefore n = 0$. (⇒ Mechanism = structure).

2). 


Four bar Mechanism

$L = 4$

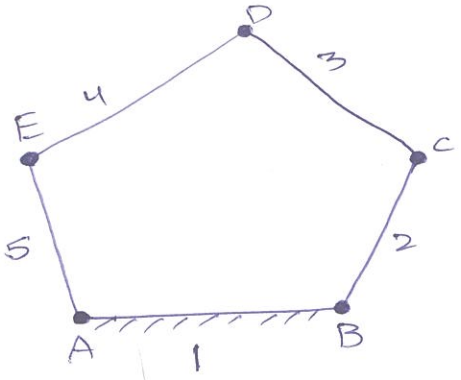
$j = 4$ BJ

$\Rightarrow n = 3(4-1) - 2(4)$

$\therefore n = 1$

(⇒ Mechanism can be driven by single i/p motion).

3).




Five bar Mechanism

$L = 5$

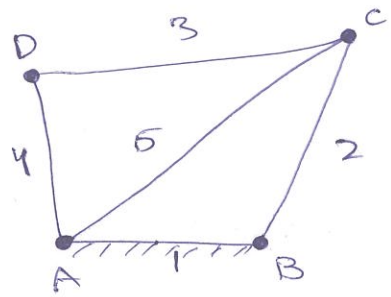
$j = 5$ BJ

$\Rightarrow n = 3(5-1) - 2(5)$

$\therefore n = 2$

($\Rightarrow$ 2 i/p motions).

4).




Five bar Mechanism

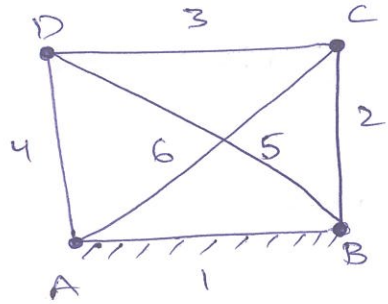
$L = 5$

$j = 6$ ($\because$ 2BJ, 2TJ = 2+2(2 =6).

$\Rightarrow n = 3(5-1) - 2(6)$

$\therefore n = 0$ ($\Rightarrow$ Structure)

5).




Six bar Mechanism

$L = 6$

$j = 8$ ($\because$ 4TJ = 2(4) = 8)

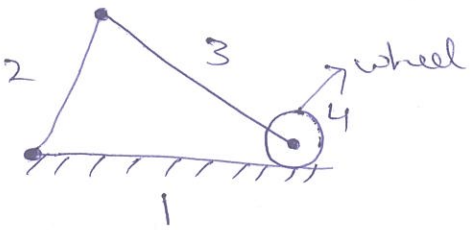
$\Rightarrow n = 3(6-1) - 2(8)$

$\therefore n = -1$

($\Rightarrow$ statically indeterminate structure).

$\Rightarrow n = 3(L-1) - 2j - h$

*




$L = 4$ , $h = 1$

$j = 3$ BJ

$\Rightarrow n = 3(4-1) - 2(3) - 1$

$\therefore n = 2$

## * Grubler's criterion for plane mechanisms:-

It is applicable if $n=1$, so -

$\Rightarrow \quad n = 3(l-1) - 2j - h$

$\Rightarrow \quad 1 = 3(l-1) - 2j - 0 \quad (\because n=1, h=0)$

$\Rightarrow \quad 1 = 3l - 3 - 2j$

$\therefore 3l - 2j - 4 = 0$

This eqn is called Grubler's criterion.

## * Inversion of Mechanism:-

→ By fixing different links in a kinematic chain, we can obtain different mechanisms.

→ This method is called "inversion of mechanism".

## * Types of kinematic chains:-

1). Four bar chain (or) quadric cyclic chain.

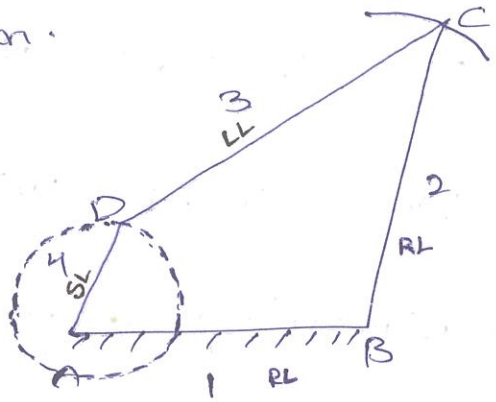
2). Single slider crank chain

3). Double slider crank chain.

(These chains consist of mostly lower pairs only)

## 1) Four bar chain or Quadric cycle chain :-

→ Simplest & basic kinematic chain is a four bar chain or quadric cycle chain.




→ It consists of four links & four turning pairs (4LP).

→ According to "Grashof's law" for a four bar mechanism, sum of shortest & longest link lengths should not be greater than sum of remaining two link lengths if there is to be continuous relative motion b/w two links. ( SL + LL < RL + RL )

→ Mechanism design should be such that i/p crank makes a complete revolution relative to other links.

→ In this chain, shortest link will make a complete revolution relative to other 3 links, if it satisfies grashof's law. Such link is called "crank / driver".

→ AD (link4) is a crank.

→ Link BC (link2) which makes a partial rotation or oscillates is called "lever / rocker / follower"

→ Link CD (link 3) which connects crank & lever is called "connecting rod / coupler"

→ Fixed link AB (link 1) is called "frame".

→ When crank (l4) is driver, mechanism is transforming "rotary motion" into "oscillating motion".

* Inversions of Four Bar chain:-

1). Beam engine (crank & lever mechanism)

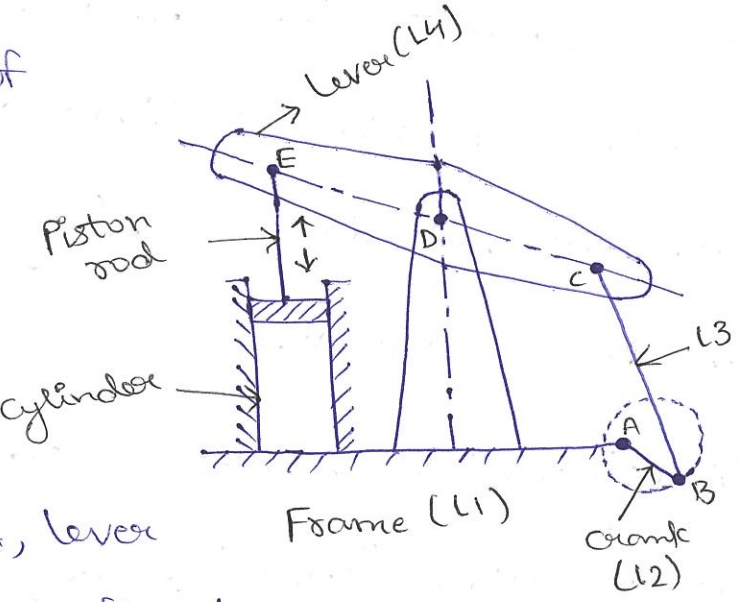
2). Coupling rod of a locomotive (Double crank mechanism)

3). Watt's indicator mechanism (Double lever mechanism).

* 1). Beam engine :-

→ It consists of 4 links.

→ In this, when crank rotates about fixed centre A, lever oscillates about fixed centre D.



→ End E of lever CDE is connected to piston rod which reciprocates due to rotation of crank.

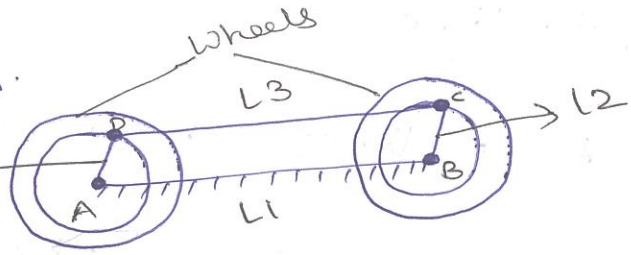
→ This mechanism is to convert rotary motion into reciprocating motion.

## * 2). Coupling rod of a locomotive :-

→ It consists of 4 links.

→ In this, AD & BC act as cranks & are connected to wheels. (∵AD & BC makes rotary complete rotation)

→ CD acts as coupling rod & AB as fixed link.



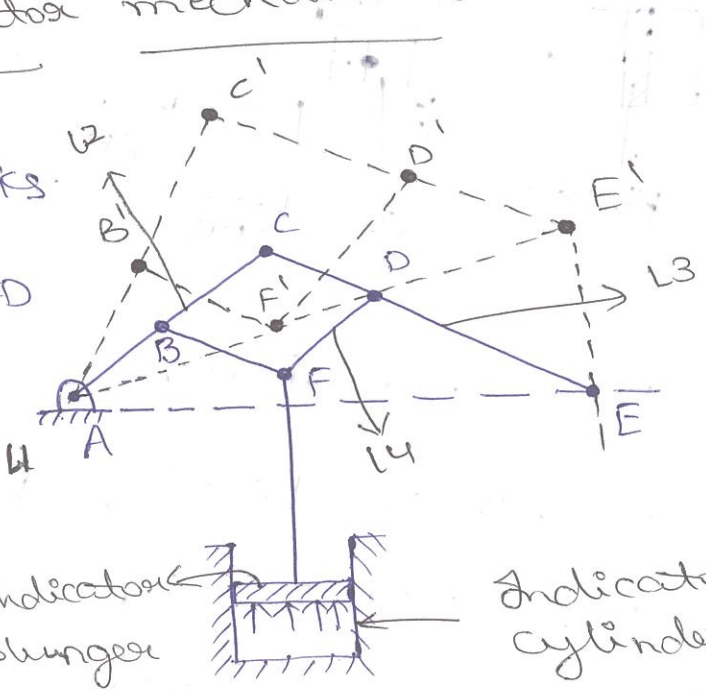

→ This mechanism is to transmit rotary motion from one wheel to other wheel.

## * 3). Watt's indicator mechanism :-

→ It consists of 4 links.

→ In this CE & BFD act as levers & L1 is fixed link.

→ Displacement indicator of link BFD is plunger

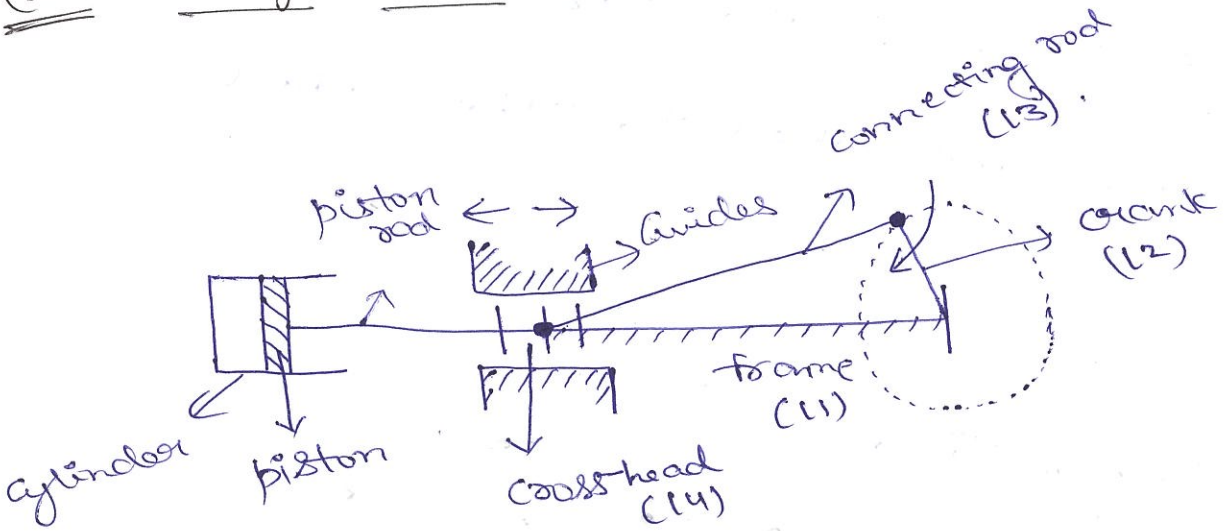


directly proportional to pressure of gas or steam which acts on plunger.

→ On any small displacement of mechanism, tracing pt. E traces out approximately a st. line.

→ Dark lines shows the initial position & dotted lines shows the position of mechanism when gas/steam pressure acts on plunger.

## (II). Single slider crank chain :-




→ It is a modification of four bar chain.

→ It consists of 1 sliding pair & 3 turning pairs.

→ Links 1 & 2, 2 & 3, 3 & 4 form 3 turning pairs & links 4 & 1 form a sliding pair.

→ L1 is fixed link, L2 is crank, L3 is connecting rod, L4 is cross-head.

→ As crank rotates, cross-head reciprocates in the guides & thus piston reciprocates in the cylinder.

→ In this rotary motion is converted into reciprocating motion & vice versa.

→ Eg:- Reciprocating steam engine mechanism.

* Inversions of single slider crank chain:-

1). Pendulum pump or Bull engine.

2). Oscillating cylinder engine

3). Rotary internal combustion engine or Gnome engine.

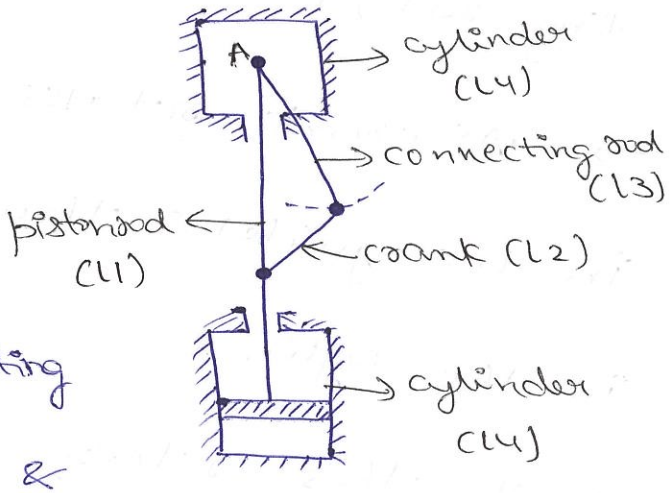
4). Crank & slotted lever quick return motion mechanism.

5). Whitworth quick return motion mechanism.

* 1). Pendulum pump :-

→ It consists of 4 links.

→ In this link 4 (cylinders) are fixed links.



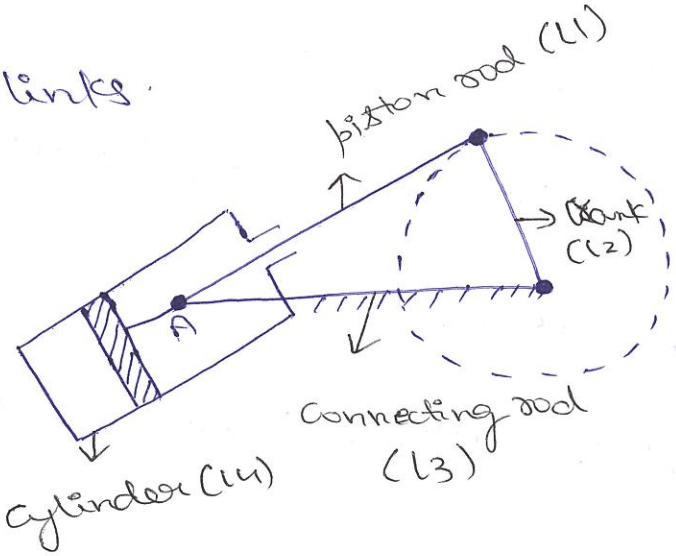

→ When crank rotates, connecting rod oscillates about pin at A & piston attached to piston rod reciprocates.

→ In this, rotary motion is converted into reciprocating motion. Eg:- Duplex pump.

* 2). Oscillating cylinder engine :-

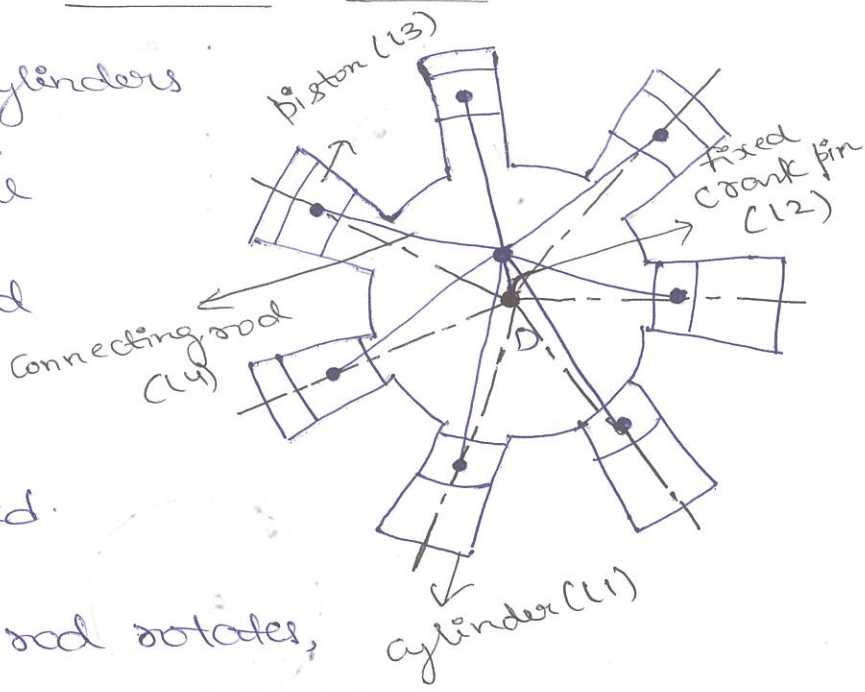
→ It consists of 4 links.

→ When crank rotates, piston attached to piston rod reciprocates & cylinder oscillates about pin at A.




→ In this rotary motion is converted into reciprocating & then to Oscillatory motion.

*3) Rotary internal combustion engine :-




→ It consists of 7 cylinders in one plane & all revolves about fixed centre D, while crank (12) is fixed.

→ When connecting rod rotates, piston reciprocates inside the cylinders.

→ In this, rotary is converted into reciprocating.

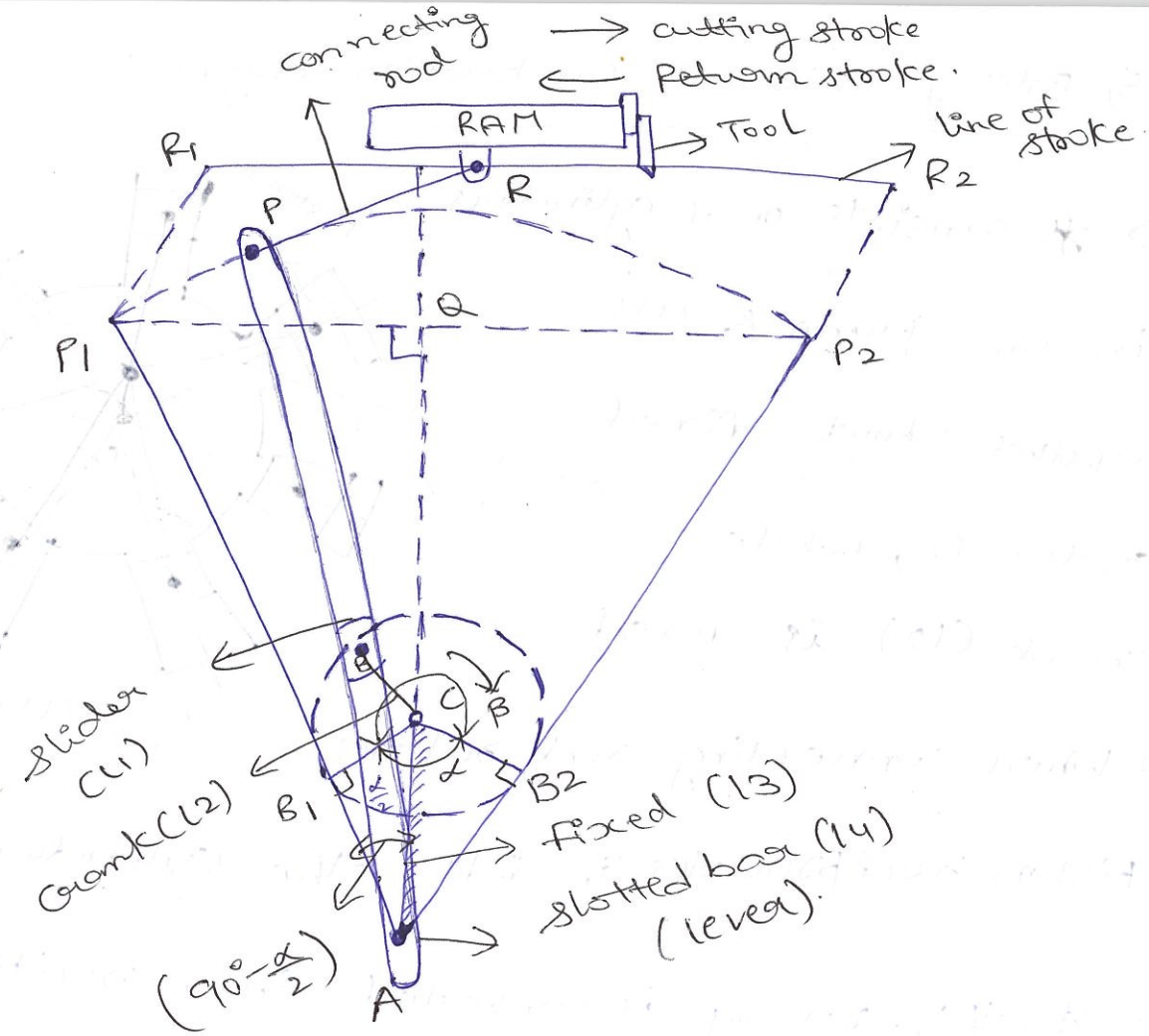
Eg :- Gas turbines.

*4). Crank & slotted lever quick return motion mechanism :-

→ When crank rotates about centre 'C', slider in the slotted bar(AP) slides. ~~AP~~

→ When slider slides, AP ~~slotted~~ bar oscillates about pt. A.

→ Then (PR) connecting rod transmits motion from AP to ram which carries tool & reciprocates along line of stroke $R_1R_2$.




→ line of stroke of ram ($R_1R_2$) is ⊥ to AC.

→ $AP_1$ & $AP_2$ are tangential to circle

→ Forward cutting stroke occurs when crank rotates from $CB_1$ to $CB_2$ (angle β) in C.Direction.

→ Return stroke occurs when crank rotates from $CB_2$ to $CB_1$ (angle α) in ∠ACD.

→ $$\frac{\text{Time of CS}}{\text{Time of RS}} = \frac{\beta}{\alpha} = \frac{\beta}{360^\circ - \beta} \text{ (or) } \frac{360^\circ - \alpha}{\alpha}.$$

→ Tool travels from $R_1$ to $R_2$.

→ length of stroke $= R_1R_2 = P_1P_2$

$= 2P_1Q = 2AP_1 \sin\angle P_1AQ$

$\left(\because \sin\angle P_1AQ = \frac{P_1Q}{AP_1} \text{ (from } \Delta le\ AQP_1)\right.$

$\Rightarrow P_1Q = AP_1 \sin(P_1AQ)$

$= 2AP_1 \sin\left(90^\circ - \frac{\alpha}{2}\right)$

$= 2AP \cos\frac{\alpha}{2} \quad (\because AP_1 = AP)$

$= 2AP \times \frac{CB_1}{AC}$

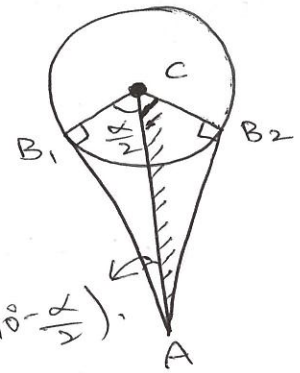
$\left(\because \text{from } \Delta le\ ACB_1, \cos\frac{\alpha}{2} = \frac{CB_1}{AC}\right).$

$= 2AP \times \frac{CB}{AC} \quad (\because CB_1 = CB).$

$\therefore$ length of stroke $= 2AP \times \frac{CB}{AC}$.

* A crank & slotted lever mechanism used in a shaper has a centre distance of 300 mm between centre of oscillation of slotted lever & centre of rotation of the crank. The radius of crank is 120mm. Find ratio of time of cutting to time of return stroke.

→ $AC = 300mm$, $CB_1 = 120mm$,




$$\sin A = \frac{CB_1}{AC}$$

$$\sin\left(90^\circ - \frac{\alpha}{2}\right) = \frac{120}{300} = 0.4$$

$$\sin\left(90^\circ - \frac{\alpha}{2}\right) = 0.4$$

$$90^\circ - \frac{\alpha}{2} = \sin^{-1}(0.4) = 23.6^\circ$$

$$\frac{\alpha}{2} = 90^\circ - 23.6^\circ$$

$$\therefore \alpha = 132.8^\circ$$

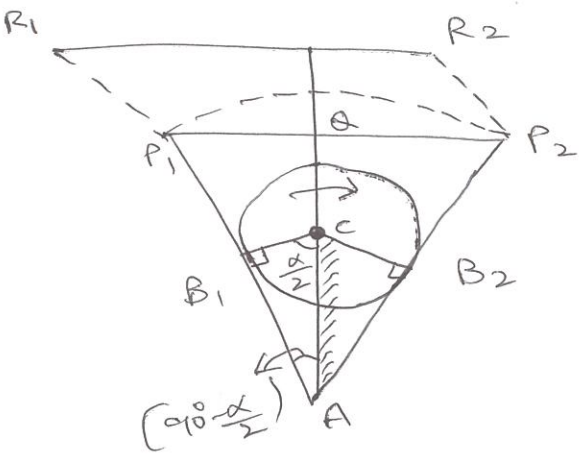
$$\Rightarrow \frac{\text{Time of C.S}}{\text{Time of R.S}} = \frac{360^\circ - \alpha}{\alpha} = 1.72$$

* In a crank & slotted lever quick return motion mechanism, distance b/w fixed centres is 240 mm & length of driving crank is 120 mm, find inclination of slotted bar with vertical in extreme position & time ratio of cutting stroke to return stroke.

If length of slotted bar is 450 mm, find length of stroke if line of stroke passes through extreme positions of free end of lever.

→ AC = 240mm,

$CB_1 = 120\,mm$, $AP_1 = 450mm$.




⇒ from Δle $ACB_1$

$$\sin A = \frac{CB_1}{AC}$$

$$\sin A = \frac{120}{240} = 0.5$$

$\therefore$ Inclination, $A = \sin^{-1}(0.5) = 30^\circ$ ✓

⇒ $\sin A = 0.5$

$$\sin\left(90^\circ - \frac{\alpha}{2}\right) = 0.5$$

$$90^\circ - \frac{\alpha}{2} = \sin^{-1}(0.5) = 30^\circ$$

$$\frac{\alpha}{2} = 90^\circ - 30^\circ = 60^\circ$$

$\therefore \alpha = 120^\circ$

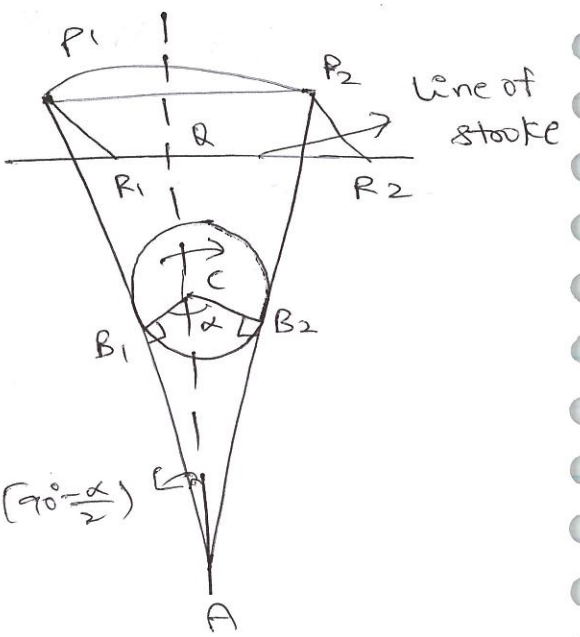
$$\therefore \frac{\text{Time of a.s}}{\text{Time of Re.s}} = \frac{360^\circ - \alpha}{\alpha} = \frac{360^\circ - 120^\circ}{120^\circ} = 2$$ ✓

⇒ length of ~~stroke~~ = $R_1R_2$

$$= 2 \times \frac{AP \times CB}{AC} = \frac{2 \times 450 \times 120}{240}$$

$\therefore L = 450mm$ ✓

* Fig. shows layout of a quick return mechanism of the oscillating link type, for a special purpose m/c. The driving crank BC is 30mm long & time ratio of working stroke to return stroke is to be 1.7. If length of stroke is 120mm, determine dimensions of AC & AP.




→ BC = 30mm, $R_1R_2$ = 120mm,

Time ratio = 1.7.

$\Rightarrow 1.7 = \frac{360-\alpha}{\alpha}$

$\therefore \alpha = 133.3°$.

$\Rightarrow \frac{\alpha}{2} = 66.65°$.

From $\Delta AB_1C$, $\sin\left(90°-\frac{\alpha}{2}\right) = \frac{CB_1}{AC}$

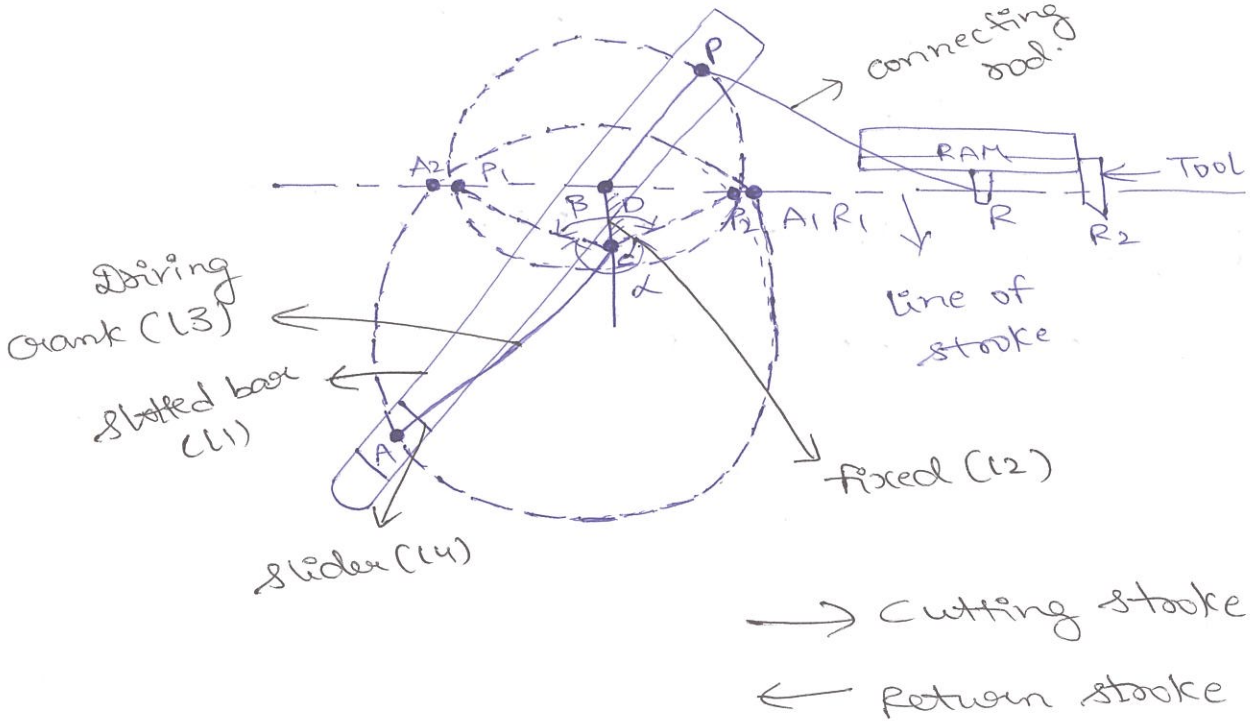
$\therefore \Rightarrow AC = \frac{CB_1}{\sin(90°-66.65°)} = \frac{30}{0.3963} = 75.7mm$ ✓

length of stroke, $L = R_1R_2 = 2 \times AP \times \frac{CB}{AC}$.

$\Rightarrow AP = \frac{R_1R_2 \times AC}{2 \times CB} = \frac{120 \times 75.7}{2 \times 30}$

$\therefore AP = 151.4mm$ ✓

## * 5). Whitworth quick return motion mechanism:




→ It is same lyk 4th mechanism, when crank rotates, slider slides in the slotted bar.

→ Then slotted bar oscillates at pivoted pt. D & the it transmits motion to connecting rod & then to tool.

→ When cutting stroke occurs when crank rotates from $CA_1$ to $CA_2$ through an angle $\alpha$ in CD.

→ Return stroke occurs when crank rotates from $CA_2$ to $CA_1$ through an angle $\beta$ in CD.

→ $\frac{\text{Time of cutting stroke}}{\text{Time of return stroke}} = \frac{\alpha}{\beta}$

$= \frac{\alpha}{360° - \alpha}$ (or) $\frac{360° - \beta}{\beta}$.

→ Length of stroke, $L = R_1R_2$ (or) $2PD$.

(& $PR = P_1R_1 = P_2R_2$).

* In a whitworth quick return motion mechanism, as shown in fig, the distance b/w fixed centers is 50mm & length of driving crank is 75mm. The length of slotted lever is 150mm & length of c.rod is 135mm. Find ratio of time of C.S to R.S. &

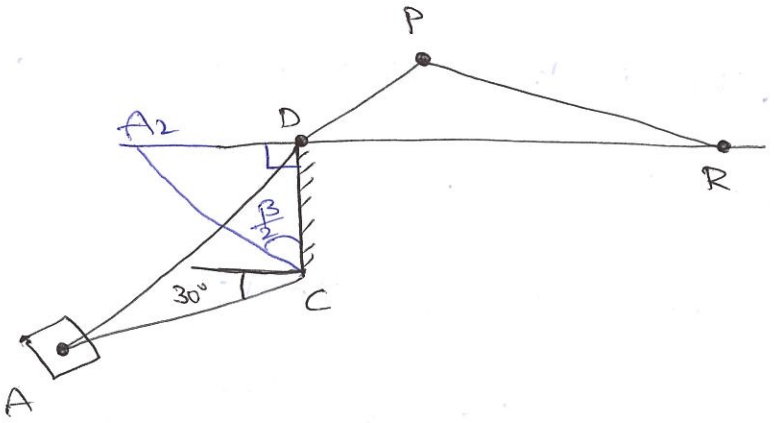
→ CD = 50mm,

CA = 75mm,

AP = 150mm,

PR = 135mm,

T.R = ?




From $\Delta le$ $CDA_2$, $\cos\frac{\beta}{2} = \frac{CD}{CA_2} = \frac{50}{75} = 0.667$.

$\frac{\beta}{2} = \cos^{-1}(0.667) = 48.2°$

$\therefore \beta = 96.4°$.

$$\therefore \text{Time Ratio} = \frac{360-\beta}{\beta}$$

$$= \frac{360-96.4}{96.4} = 2.735 \checkmark$$

*(III). Double slider crank chain :-

→ A kinematic chain which consists of 2 T.P's & 2 S.P's is known as D.S.C.C.

→ In elliptical trammels, l1 & l3 are sliders & form S.P's with l4.

→ l2 is a bar which forms T.P with l1 & l3.

* Inversions of Double slider crank chain :-

1). Elliptical trammels.

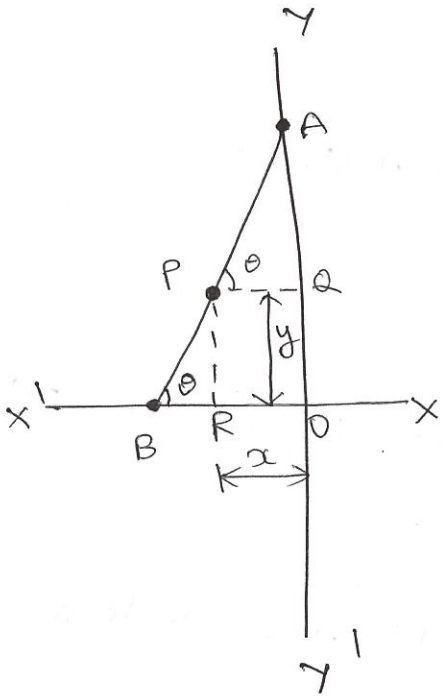
2). Scotch yoke mechanism.

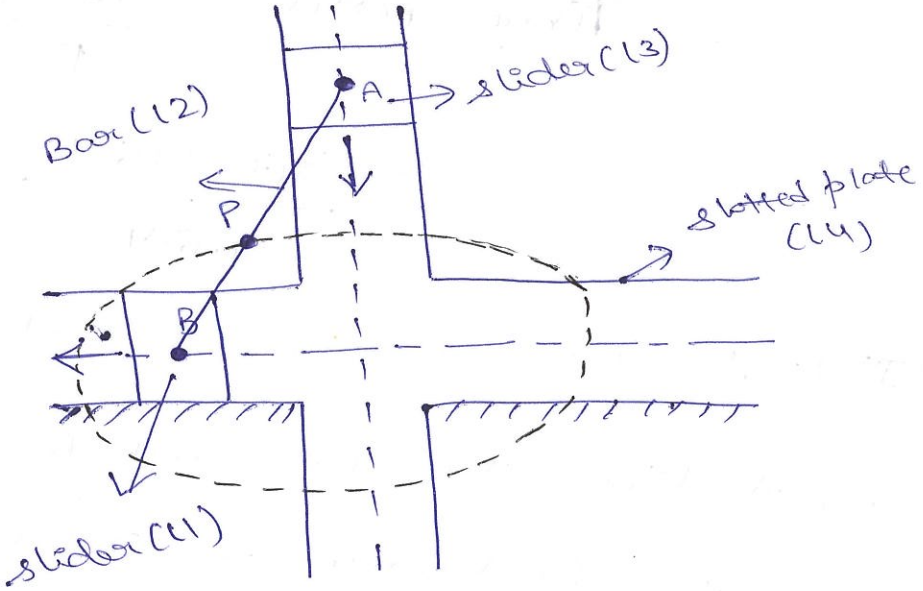
3). Oldham's coupling.

*1). Elliptical trammels :-

→ It is an instrument used for drawing ellipses.

→ link 4 is fixed plate has two straight grooves cut in it, at right angles to each other.




→ When links 1 & 3 slide along their respective grooves, any pt. on link 2 such as pt. 'P' traces out an ellipse on surface of 14.

→ Thus, from this AP = semi-major axis & BP = semi-minor axis of ellipse.

→ Proof :-

Co-ordinates of pt. P are —

$x = PQ = AP\cos\theta$. $\left(\because \cos\theta = \frac{PQ}{AP}, \Delta le\ APQ\right)$

& $y = PR = BP\sin\theta$. $\left(\because \sin\theta = \frac{PR}{BP}, \Delta le\ PBR\right)$.

$\Rightarrow \frac{x}{AP} = \cos\theta$ & $\frac{y}{BP} = \sin\theta$.

By squaring & adding, we get —

$$\frac{x^2}{(AP)^2} + \frac{y^2}{(BP)^2} = \cos^2\theta + \sin^2\theta$$

$$\therefore \frac{x^2}{(AP)^2} + \frac{y^2}{(BP)^2} = 1.$$

⇒ This eqn is called ellipse. ('P' traces ellipse)

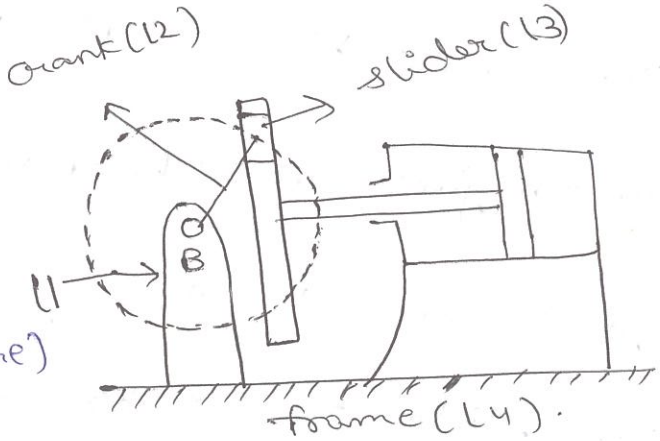
→ If 'P' is mid pt of AB, then AP = BP,

then eqn is $\frac{x^2}{(AP)^2} + \frac{y^2}{(AP)^2} = 1.$

$$\therefore x^2 + y^2 = (AP)^2$$

⇒ This eqn is circle of radius AP. ('P' traces circle)

## * 2). Scotch yoke mechanism:-

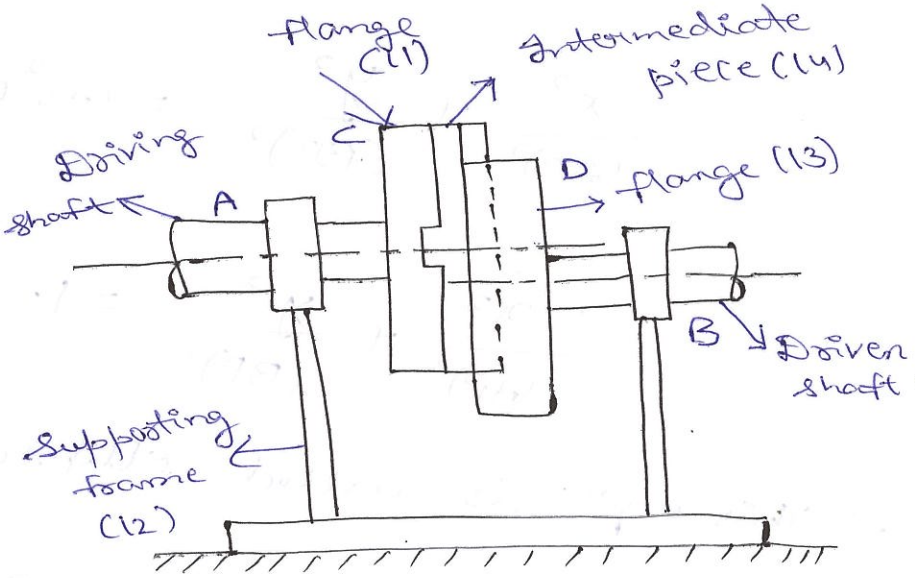



→ Here, 11 is fixed.

→ When crank(12) rotates, link 4 (frame) reciprocates.

→ It is used to convert rotary motion into reciprocating motion.

## *3). Oldham's coupling:-




→ It is used for connecting 2 shafts whose axes are at a small distance apart.

→ These shafts are coupled in such a way that if one shaft rotates, other shaft also rotates at same speed.

→ Two flanges L1 & L3 are connected to 2 shafts.

→ These flanges have diametrical slots cut in their inner faces.

→ Intermediate piece (L4) which is circular disc, have two tongues on each face at rt angles.

→ Tongues on L4 closely fit into slots & L4 can slide or reciprocate in those slots.

→ When driving shaft rotates, L1 causes L4 to rotate & further rotates L3 & thus driven shaft rotates.

→ Hence 11, 13, 14 have same angular velocity ($\omega$)

→ Max. sliding speed of each tongue (m/s)

$$\Rightarrow V = \omega d.$$

When $V$ = Max. sliding speed (m/sec).

$\omega$ = Angular velocity of each shaft (rad/s)

$d$ = Distance b/w axes of shafts (m).

UNIT-II

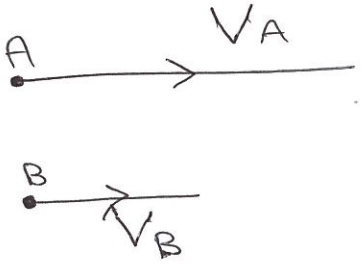
# Kinematics — ①

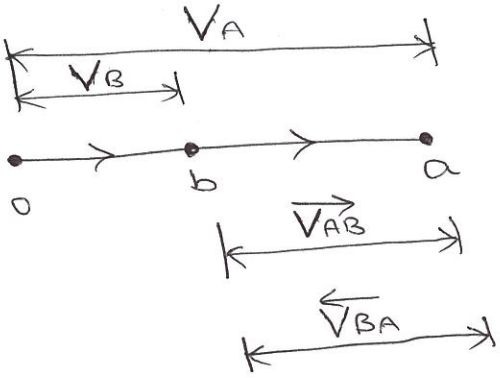
## * Relative Velocity Method :-

To determine the velocity of different points in the mechanism, this method is used.

To determine acceleration, velocity analysis is imp.

## * Relative velocity of two bodies moving in st. lines :-




consider two bodies A & B moving along || lines with absolute velocities $V_A$ & $V_B$ & $V_A > V_B$,

then relative velocity of A wrt B —

$V_{AB}$ = Vector diff. of $V_A$ & $V_B$

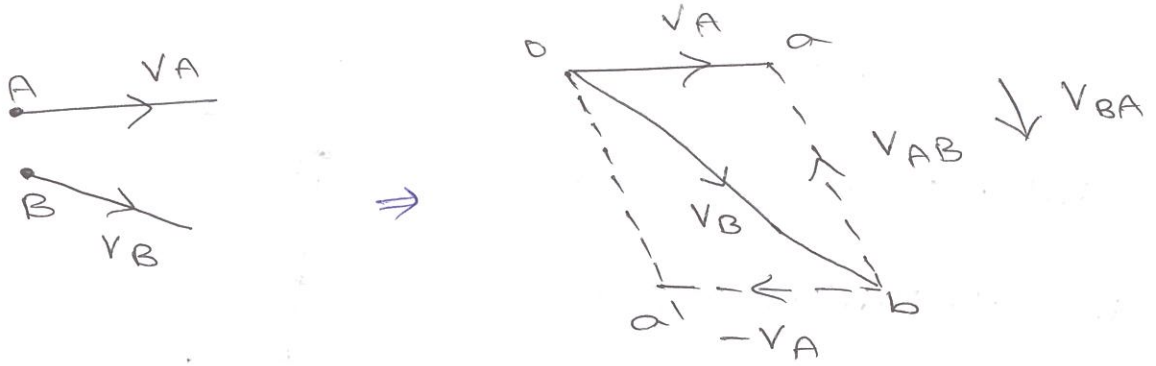
$\therefore V_{AB} = \overline{V_A} - \overline{V_B}$

In vector form, $\overline{ba} = \overline{oa} - \overline{ob}$

||ly, relative velocity of B wrt A —

$\therefore V_{BA} = \overline{V_B} - \overline{V_A}$ & $\overline{ab} = \overline{ob} - \overline{oa}$

## * In Inclined lines :-




Relative velocity is obtained by 11gram law of velocity (or) triangle law of velocity. For this mark a pt 'o' & from o, draw vector oa to represent $V_A$ & ob to represent $V_B$. Then vector ba represents relative velocity of A wrt B ($V_{AB}$).

$$\therefore V_{AB} = \overline{V_A} - \overline{V_B}$$

$$\& \ \overline{ba} = \overline{oa} - \overline{ob}$$

$$\text{And } V_{BA} = \overline{V_B} - \overline{V_A}$$

$$\& \ \overline{ab} = \overline{ob} - \overline{oa}$$

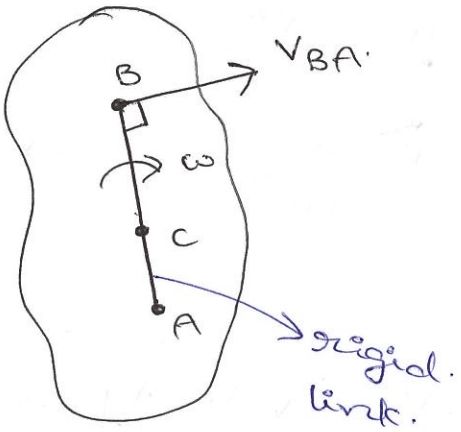
conclusion :- $V_{AB}$ & $V_{BA}$ are equal in magnitude but opp. in direction.

$$\therefore V_{AB} = -V_{BA}$$

$$\& \ \overline{ba} = -\overline{ab}$$

## * Motion of a link :-

Consider two pts. A & B on rigid link. B move relative to A in C.W.D with angular velocity 'ω'. Relative motion of B wrt A must be ⊥ to AB.




⇒ Velocity of any pt. on a link w.r. to another pt. on the same link is always ⊥ to line joining these points on the configuration.

$\therefore V_{BA} \perp AB$.

$\Rightarrow V_{BA} = \overline{ab} = \omega AB \rightarrow (1)$.

$\Rightarrow V_{CA} = \overline{ac} = \omega AC \rightarrow (2)$.

From (1) & (2),

$$\frac{V_{CA}}{V_{BA}} = \frac{\overline{ac}}{\overline{ab}} = \frac{\omega AC}{\omega AB} = \frac{AC}{AB}$$

$$\therefore \frac{V_{CA}}{V_{BA}} = \frac{AC}{AB}.$$

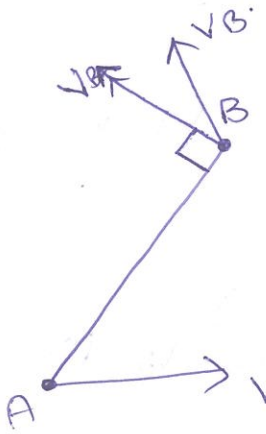
* Velocity of a point on a link by relative velocity method :-

→ consider 2 pt's A & B on a link.

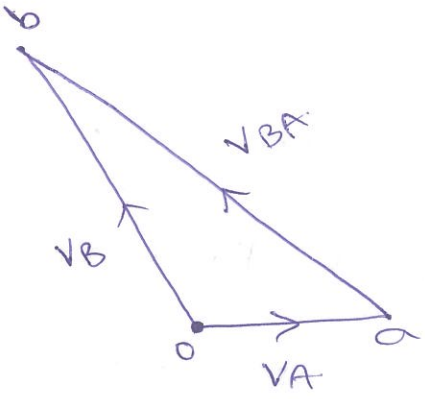
→ let $V_A$ magnitude & direction is known & $V_B$ only direction.

→ $V_B$ magnitude is determined from velocity dgm.

→ Velocity dgm steps :-




Motion of pts on link.




Velocity dgm.

1). Take point o, known as the pole.

2). Through o, draw oa ∥ & equal to $V_A$ ($\because oa \parallel = V_A$)

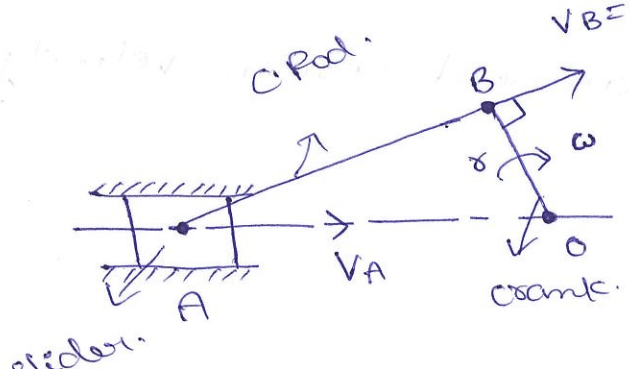
3). Through a, draw a line ⊥ to AB. This line represents $V_{BA}$ ($\because L \perp AB$).

4). Through o, draw a line ∥ to $V_B$ which intersects line of $V_{BA}$ at b. ($\because L \parallel V_B$).

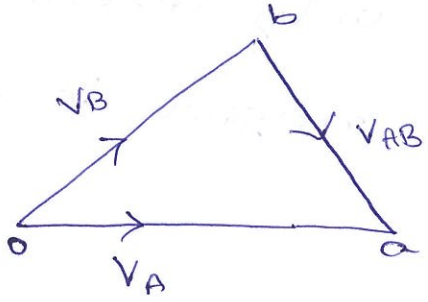
5). Measure ob, which gives $V_B$.

## * Velocities in slider crank Mechanism:-

→ Slider A is attached to C.Rod & crank rotates with an angular velocity 'ω' in C.W.D.




Slider crank mechanism.




Velocity dgm.

→ So, we can calculate $V_B$ i.e $V_B$ magnitude & direction is known.

→ Slider reciprocates along OA, so its direction ($V_A$) is known.

→ Velocity dgm steps:-

1). From o, draw ob ⊥ to OB such that

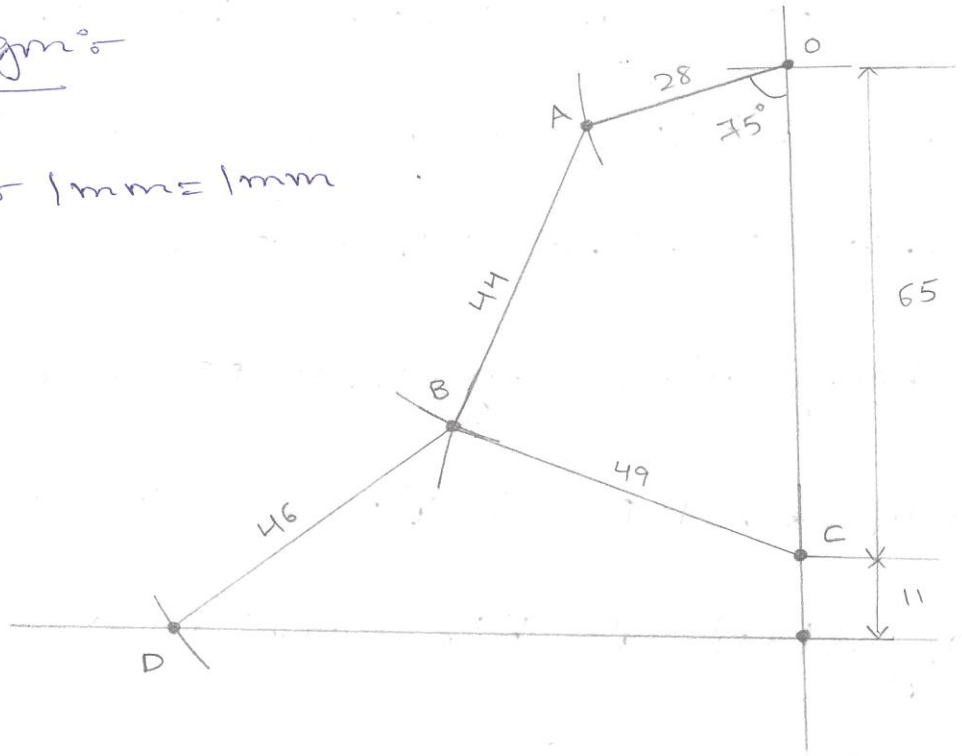
$ob = V_B$ / $V_{AB} \neq \omega r$.

2). Draw ba ⊥ to AB, which represents $V_{AB}$.

3). Draw oa ∥ to AO, which represents $V_A$ & intersects at a.

Space Dgm :-

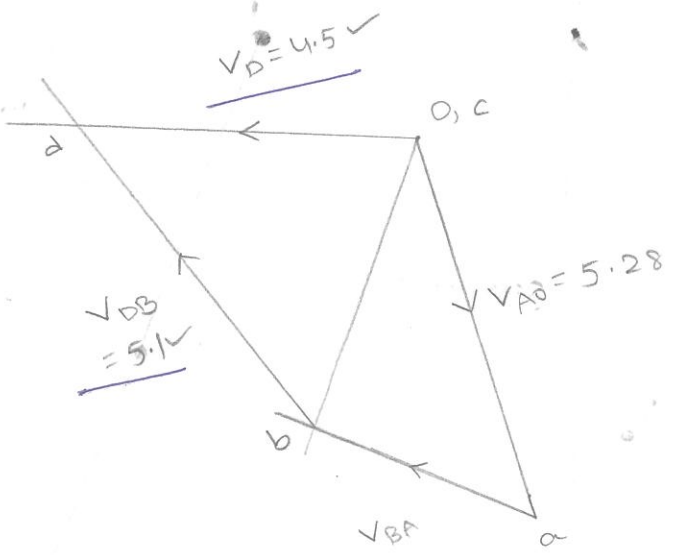
Scale :- 1mm = 1mm




Velocity Dgm :-

Scale :-

1m = 3cm



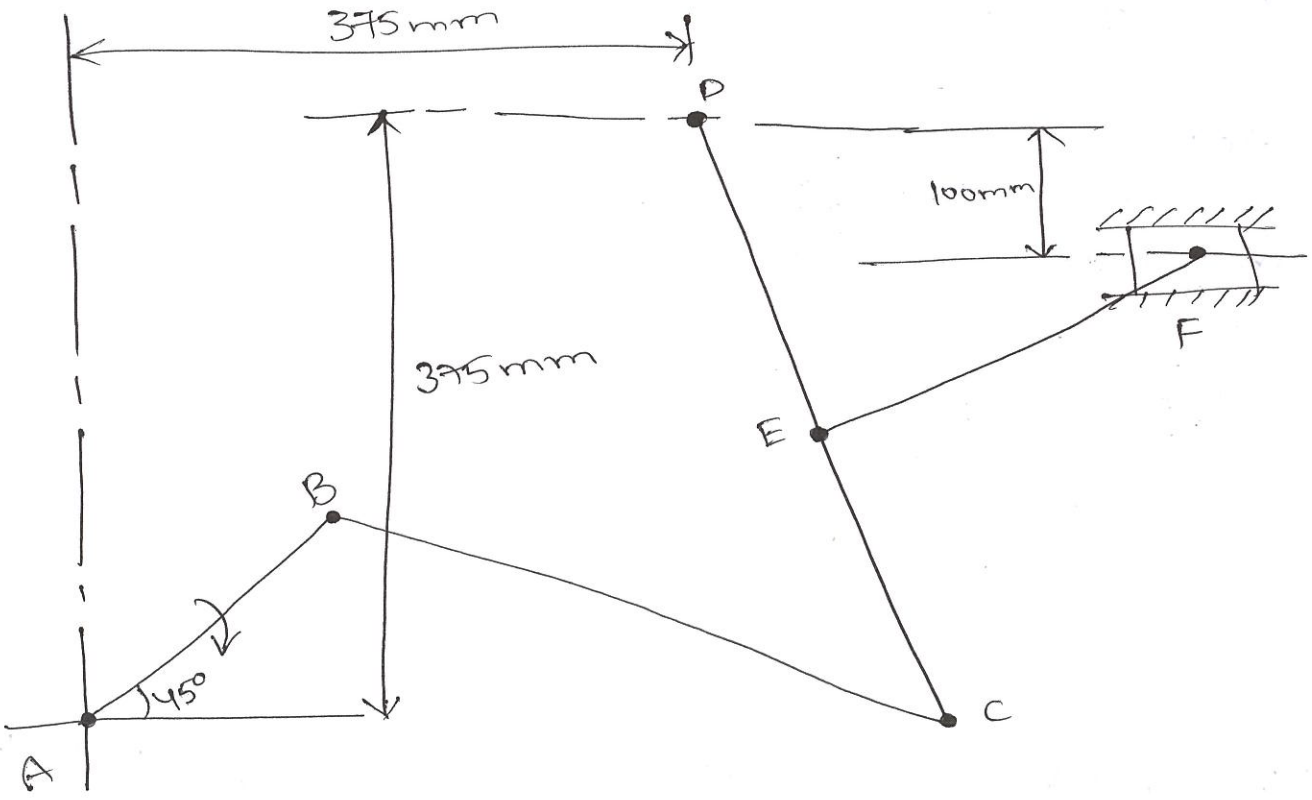

* The mechanism, as shown in fig. has the dimensions of various links as follows :-

AB = DE = 150mm ; BC = CD = 450mm , EF = 375mm

The crank AB makes an angle of 45° with horizontal & rotates about A in CWD at a

uniform speed of 120rpm. The lever DC oscillates about fixed pt. D, which is connected to AB by coupler BC. The block F moves in hori. guides, being driven by link EF. Determine :-

1). Velocity of block F, 2). Angular Vel. of DC & 3). Rubbing speed at pin C which is 50 mm in diameter.



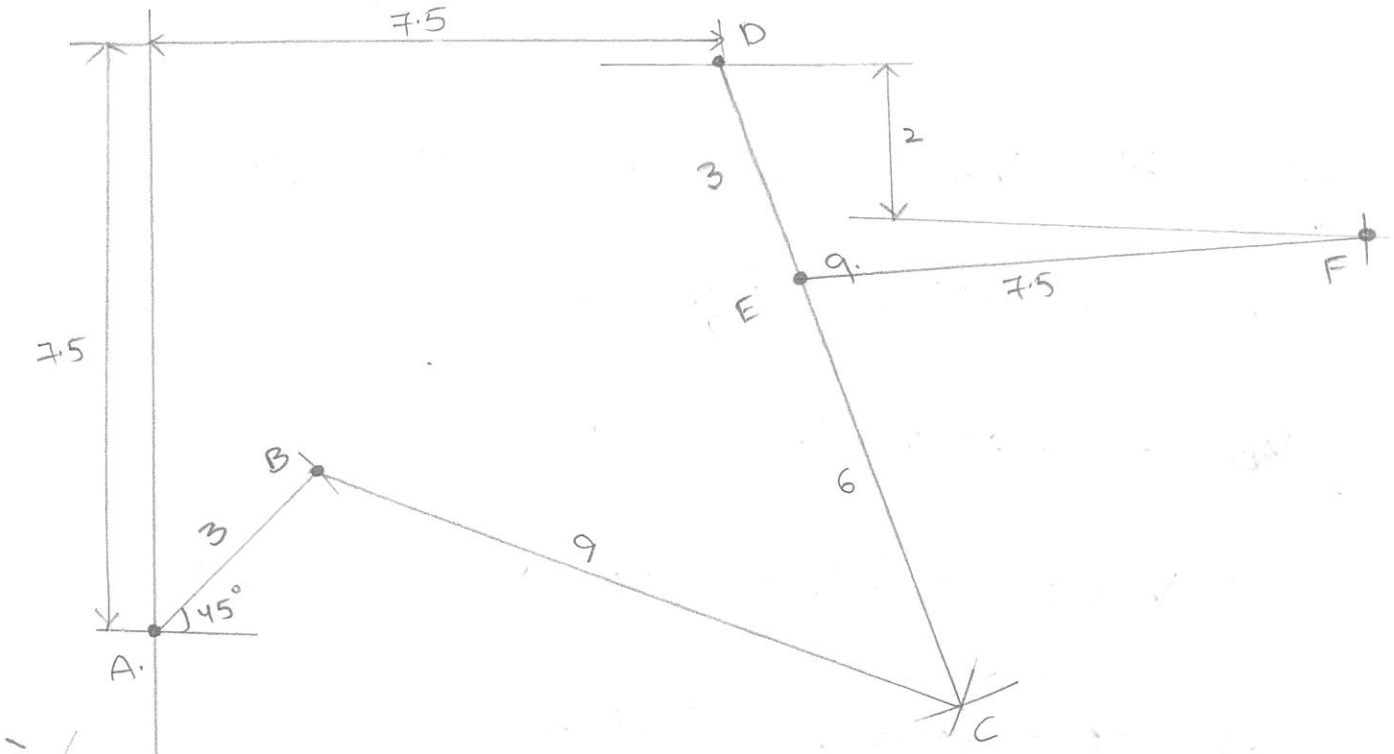

→ $N_{BA} = 120 rpm$

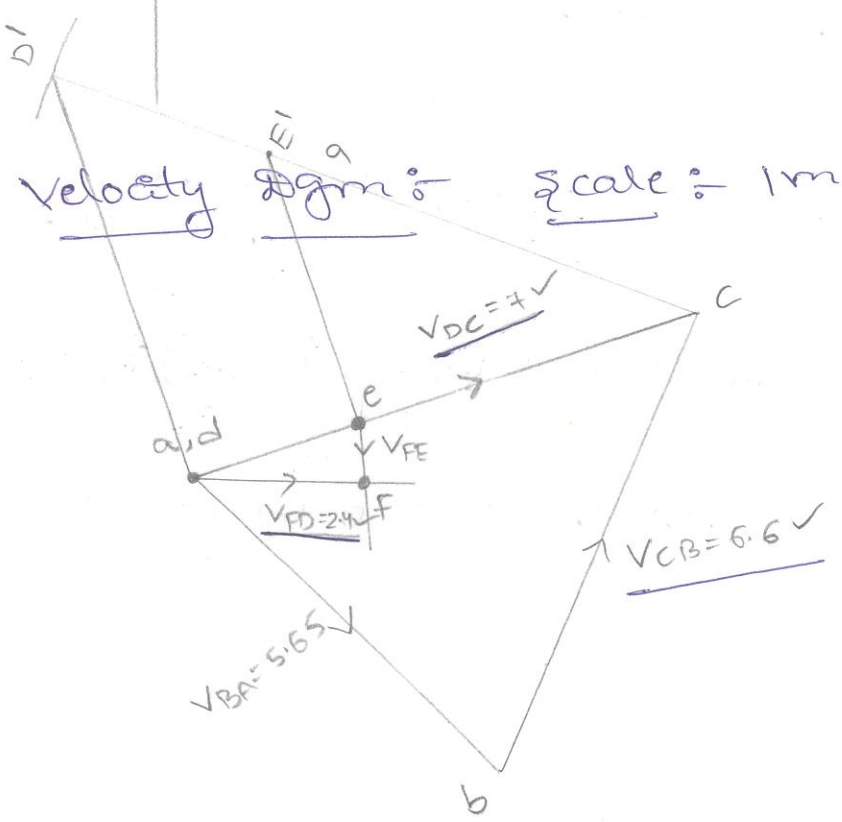
$$\omega_{BA} = \frac{2\pi N_{BA}}{60} = 4\pi \text{ rad/sec}$$

$AB = 150 mm = 0.15 m$ ∴ $V_{BA} = \omega_{BA} \times AB$

$= 1.885 m/sec.$

Space Dgm :- Scale :- 50 mm = 1 cm.




Velocity Dgm :- Scale :- 1m = 3cm.

$V_{DC} = 7$ ✓

$V_{FE}$

$V_{FD} = 2.4$ ✓

$V_{CB} = 6.6$ ✓

$V_{BA} = 5.65$ ✓

Divide, $\frac{CD}{CE} = \frac{cd}{ce}$.

1). $V_{FD} = df = 0.8$ m/sec $\left(\because df = \frac{2.4}{3} = 0.8\right)$.

2). $V_{DC} = CD = 2.33$ m/sec. $\left(\because CD = \frac{7}{3} = 2.33\right)$.

$DC = 450$ mm $= 0.45$ m

$\therefore \omega_{DC} = \frac{V_{DC}}{DC} = \frac{2.33}{0.45} = 5.185$ rad/sec. (Acw at D).

3). Given $d_c = 50mm = 0.05\ m$,

$$r_c = \frac{0.05}{2} = 0.025\ m.$$

At c pt, $V_{CB}$ & $V_{DC}$ are acting, so —

$V_{DC} = 2.33\ m/sec$, $V_{CB} = ?$

$\omega_{DC} = 5.185\ rad/sec$, $\omega_{CB} = ?$

$\Rightarrow V_{CB} = 2.2\ m/sec \left(\because cb = \frac{6.6}{3} = 2.2\right)$.

$\Rightarrow CB = 450mm = 0.45\ m$.

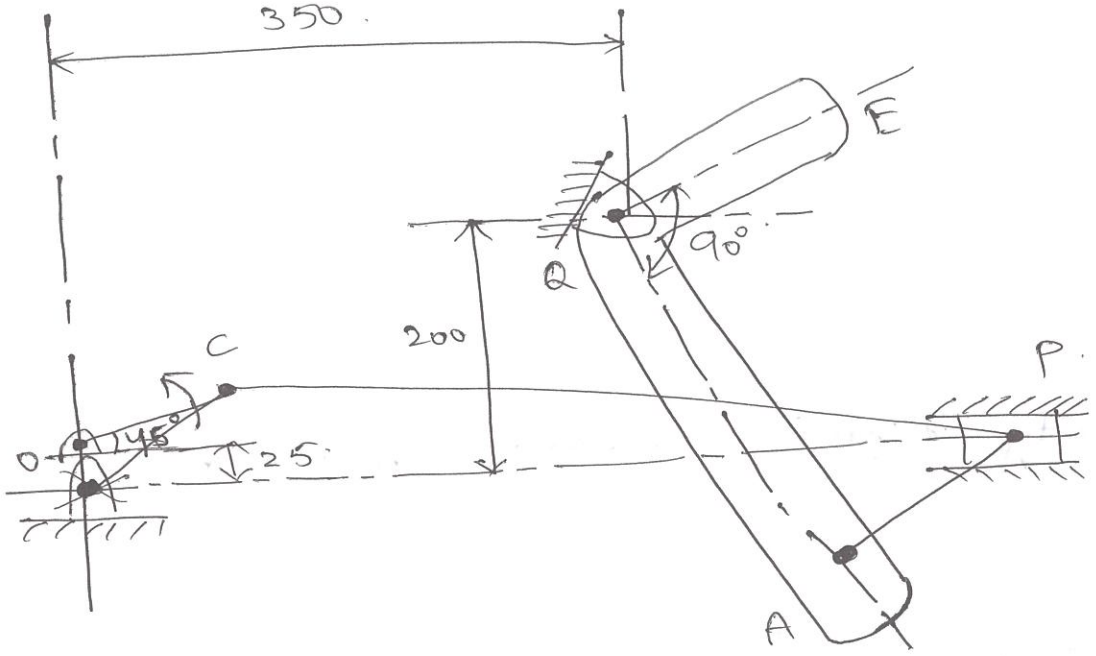
$$\therefore \omega_{CB} = \frac{V_{CB}}{CB} = \frac{2.2}{0.45} = 4.88\ rad/sec.$$
(ACW abt B).

$\therefore$ Rubbing speed at $C = (\omega_{CB} - \omega_{CD}) r_C$

$= (4.88 - 5.18)\, 0.025 = 0.0075 \simeq 0$.

* In a mechanism as shown in fig., the various dimensions are :- OC = 125 mm, CP = 500mm, PA = 125mm, AQ = 250mm, QE = 125mm. The slider P translates along an axis which is 25mm vertically below pt. O. The crank OC rotates uniformly at 1200 rpm. In anti-clockwise direction. The bell

Crank lever AQE rocks about fixed centre Q. Draw velocity dgm & calculate absolute velocity of pt. E of the lever.




→ $N_{CO} = 1200 rpm$,

$$\omega_{CO} = \frac{2\pi N_{CO}}{60}$$

$$= \frac{2\pi \times 120}{60} = 12.57 \text{ rad/sec.}$$

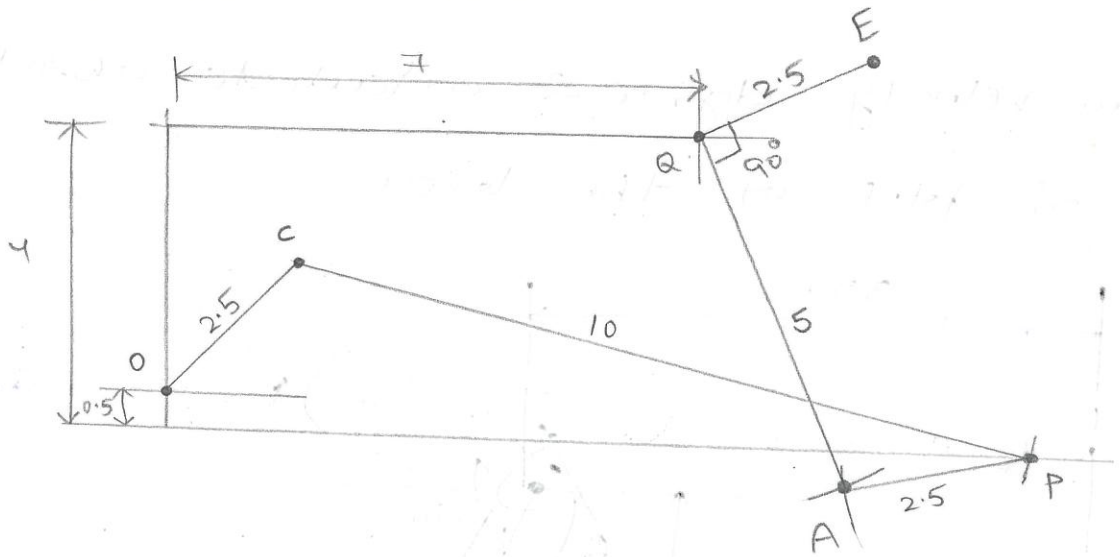
$$OC = 125mm = 0.125m.$$

$$\therefore V_{CO} = V_C = \omega_{CO} \times OC$$

$$= 12.57 \times 0.125$$

$$= 1.57 \text{ m/sec.}$$

Space Dgm :- Scale :- 50 mm = 1 cm.

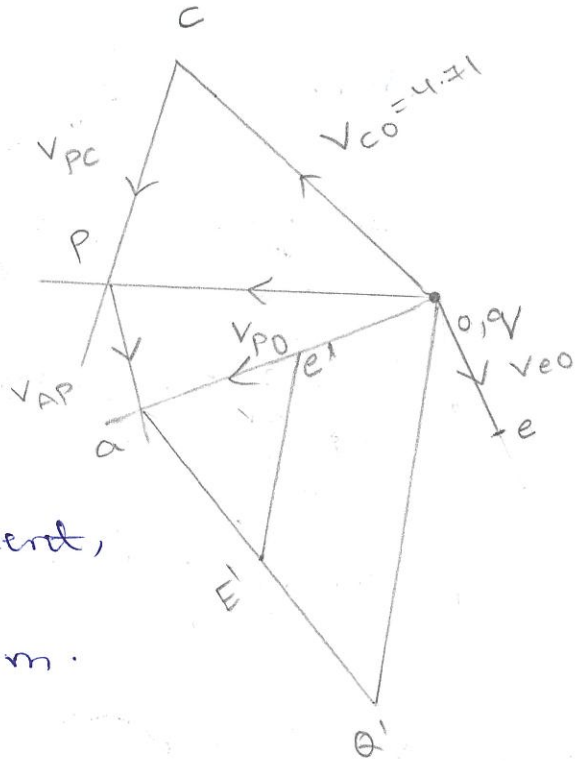



Velocity Dgm :- Scale :- 1m = 3cm.

Divide ;

$$\frac{QE}{QA} = \frac{qe}{qa}.$$

(& oe' = oe)




∴ From measurement,

$V_{EO}$ = oe = 2cm.

$= \frac{2}{3} = 0.666$ m/sec.

∴ $V_E$ = 0.66 m/sec.

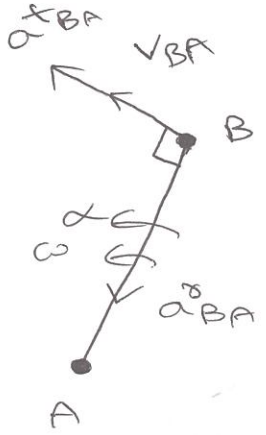
* Acceleration Dgm for a link :-

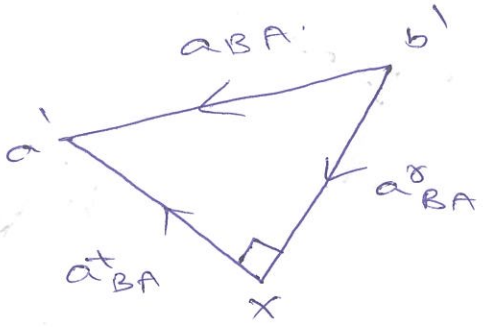
→ Rate of change of velocity wrt to time is called acceleration.

→ Acceleration has two components called -

1). Centripetal / Radial component, which is || to the rigid link. ($a^r$)

2). Tangential component, which is ⊥ to the rigid link. ($a^t$)




→ Acc. Dgm :-

1). Mark a pt. b' & draw a line b'x || to AB, which represents $a^r_{BA}$.

2). Draw a line xa' ⊥ to $a^r_{BA}$, which represents $a^t_{BA}$.

3). Join a' & b', which represents $a_{BA}$.

→ $a^r_{BA}$ & $a^t_{BA}$ are mutually ⊥ular.

## * Acceleration of a point on a link :-

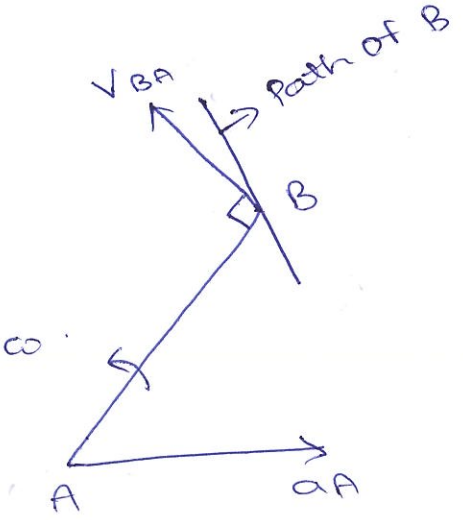
→ Consider 2 pts A & B on a rigid link.

→ Let $a_A$ magnitude & direction is known & direction of path of B i.e ($V_{BA}$) is known.

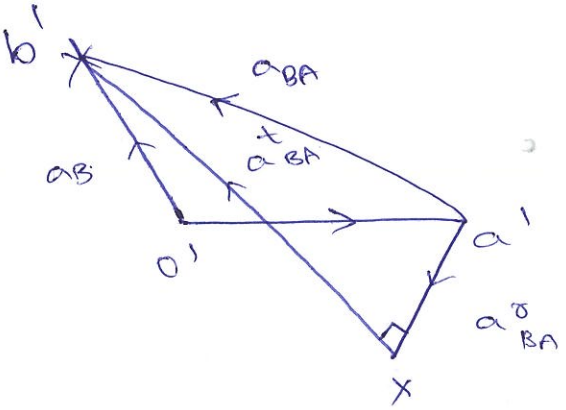
→ First calculate $V_{BA}$ & then $a^r_{BA}$ by using —

$$a^r_{BA} = \frac{V^2_{BA}}{AB}.$$

→ Acceleration Dgm steps :-




Pts. on a link




Acc. Dgm

1). Take pt. o' anywhere & draw a line o'a' with a magnitude of $a_A$ & ll to direction of $a_A$.

2). With magnitude of $a^r_{BA}$ draw a line a'x from a' & ll to AB link.

3). Then draw ⊥ line from x to AB int.

4). From o', draw a line ∥ to path of B, which intersects the previous line i.e b'.

5). ~~from~~ Join a' & b' i.e $a_{BA}$.

6). $a'x = a^r_{BA}$, $x b' = a^t_{BA}$, $o'b' = a_B$ & $a'b'$ represents $a_{BA}$.

7). Angular acceleration is obtained by -

$$\alpha_{AB} = \frac{a^t_{BA}}{AB}.$$

* Acceleration in the slider crank mechanism:-

→ consider OB crank which rotates with 'ω' in C.W.D with an angle 'θ'.

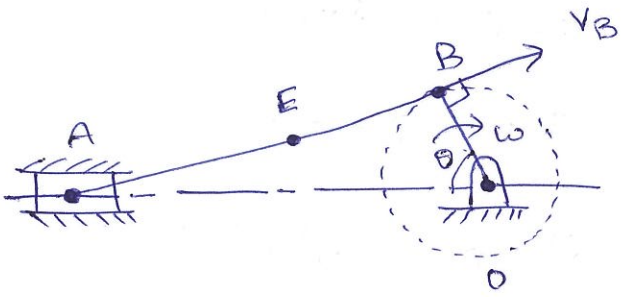
→ First calculate $V_{BO} = \omega_{BO} \times OB$ &

$$a^r_{BO} = a_B = \frac{V^2_{BO}}{OB}.$$

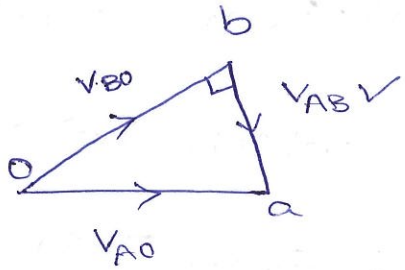
→ Next calculate $a^r_{AB}$ from velocity dgm i.e $V_{AB}$ from velocity dgm.

→ Draw velocity dgm for the mechanism.

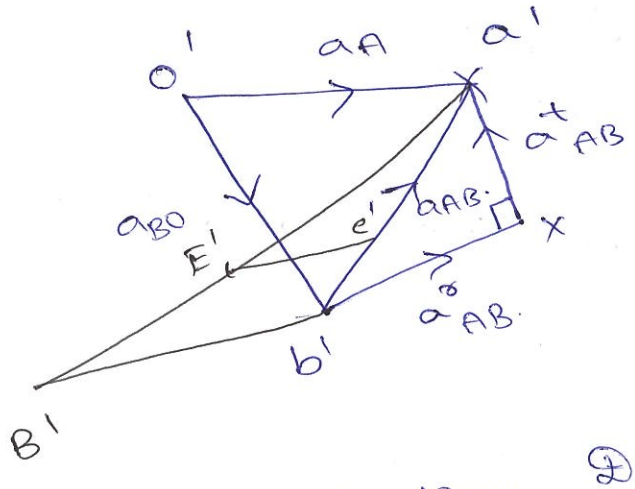
→ Acc. Dgm steps :-




slider crank mechanism




Velocity dgm.




Acceleration Dgm.

→ calculate $V_{AB}$ from Vel. dgm & then $a^r_{AB}$

by — $a^r_{AB} = \frac{V^2_{AB}}{AB}$.

1). from o', draw a line o'b' || to OB with mag. of $a_{BO}$.

2) from b', draw a line b'x || to AB with

mag. of $a^r_{AB}$.

3). Draw a ⊥ line x a' from b'x line.

4). Draw || line o'a' to OA (slider motion) which intersects previous line i.e a'.

5). Join a' & b' i.e $a_{AB}$.

6). $o'a' = a_A$, $a'b' = a_{AB}$, $x a' = a^t_{AB}$.

7). Acc. of any other pt E on C.Rod 'AB' can be obtained by —

$$\frac{a'e'}{a'b'} = \frac{AE}{AB}.$$

8). $\alpha_{AB} = \frac{a^t_{AB}}{AB}$ (CWD at B).

* The crank of a slider crank mechanism rotates clockwise at a constant speed of 300 rpm. The crank is 150mm & connecting rod is 600mm long. Determine :- 1). Linear Velocity & acc. of midpt of C.Rod, & 2). Angular Velocity & Angular acc. of C.Rod, at a crank angle of 45° from inner dead centre position.

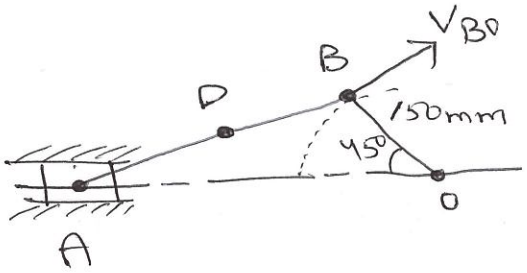
→ $N_{BO} = 300$ rpm, $\omega_{BO} = \frac{2\pi \times 300}{60} = 31.42$ rad/sec,

$OB = 150mm = 0.15\,m$, $BA = 600mm = 0.6\,m$.

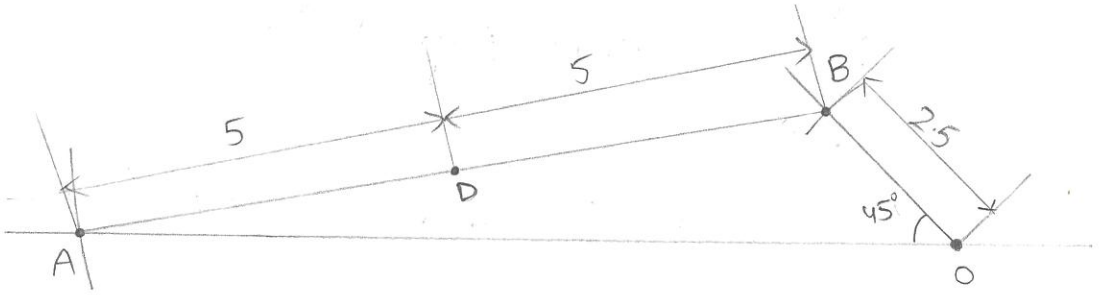
⇒ $V_{BO} = \omega_{BO} \times OB$

$= 31.42 \times 0.15$

$\therefore V_{BO} = 4.713\,m/sec.$

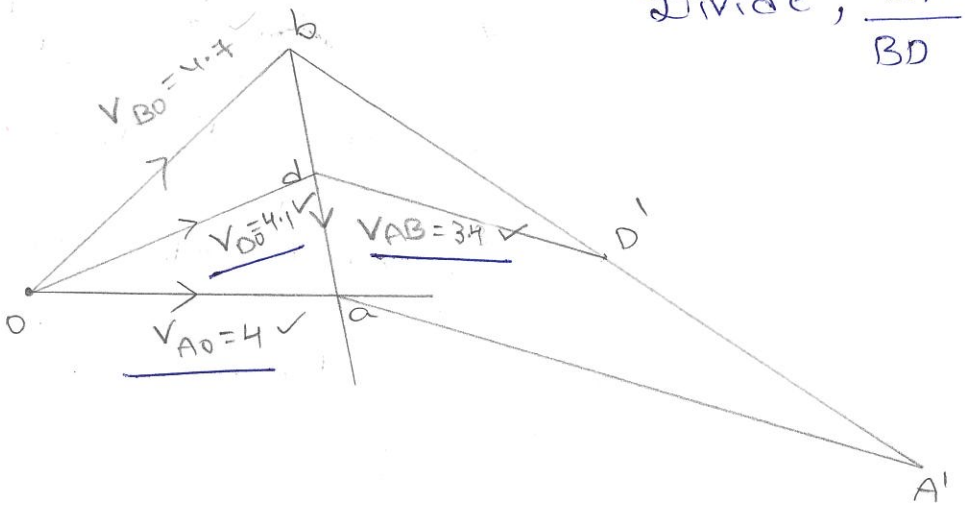



Space Dgm :- 60mm = 1cm.




Velocity Dgm :- 1m = 1cm

Divide, $\frac{BA}{BD} = \frac{ba}{bd}$



From measurement, $V_{AB} = 3.4$ m/sec &

$V_{AO} = 4$ m/sec &

$V_{DO} = 4.1$ m/sec. ✓

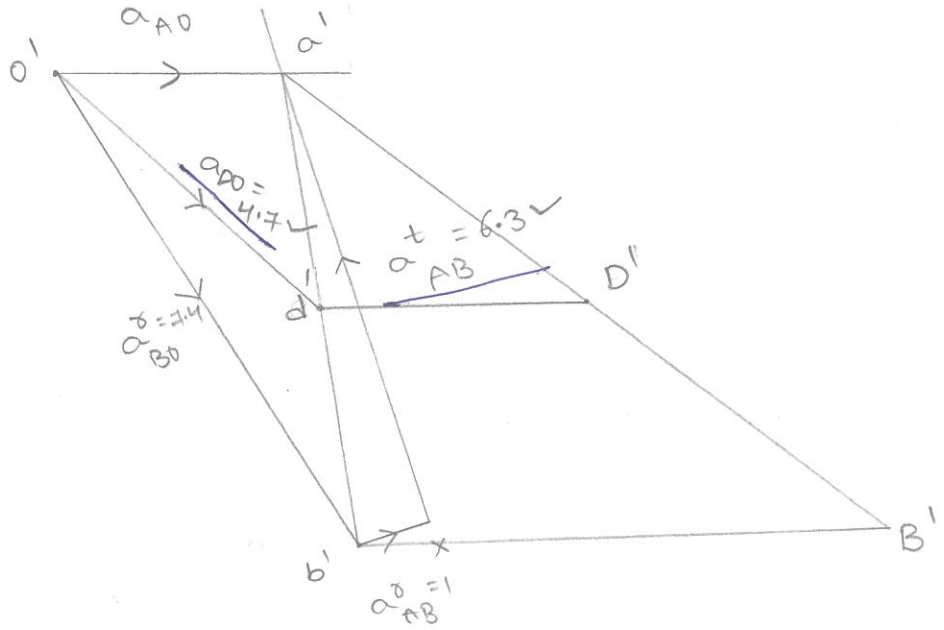
$\rightarrow$ $V_{BO} = 4.713$ m/sec, $a^r_{BO} = \frac{V_{BO}^2}{OB} = \frac{(4.713)^2}{0.15}$

$\therefore a^r_{BO} = 148.1 \text{ m/s}^2$.

$V_{AB} = 3.4$ m/sec, $a^r_{AB} = \frac{V^2_{AB}}{BA} = \frac{(3.4)^2}{0.6}$

$\therefore a^r_{AB} = 19.3 \text{ m/s}^2$.

Acceleration Diagram:- Scale :- 20m = 1cm.




From measurement, $a_{DO} = o'd' = 4.7$cm

$= 4.7 \times 20$

$\simeq 94 \text{ m/s}^2$. $\therefore a_{DO} \simeq 94 \text{ m/s}^2$ ✓

→ $\omega_{AB} = \frac{V_{AB}}{BA} = \frac{3.4}{0.6} = 5.67 \text{ rad/s}^2$ ✓

(ACW abt B).

→ $a^t_{AB} = 6.3 \text{ cm} = 6.3 \times 20 \simeq 126$ (from measurement).

⇒ $\alpha_{AB} = \frac{a^t_{AB}}{BA} = \frac{126}{0.6}$

$= 210 \text{ rad/s}^2$ ✓ (CW abt B).

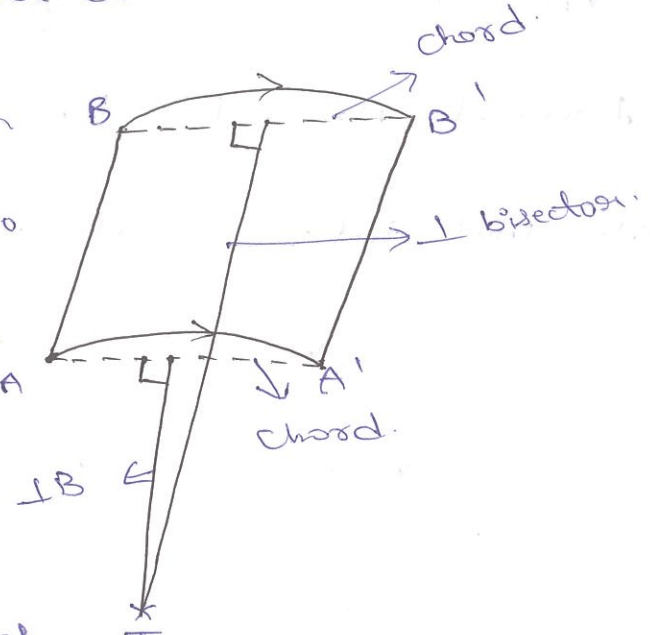
# Plane Motion of Body - (2)

* Instantaneous Centre Method :-

* Instantaneous centre of Rotation (I) :-

→ The combined motion of rotation & translation may be assumed to be motion of pure rotation about some centre I, known as "instantaneous centre of rotation" (also called centro or virtual centre).

→ Position of link goes on changing, so I position also changes. The locus of all such instantaneous centres is known as "centrode".




→ The line drawn through I & ⊥ to plane of motion is called instantaneous axis. Locus of this axis is known as "axode".

* No. of I's in a mechanism = (N)

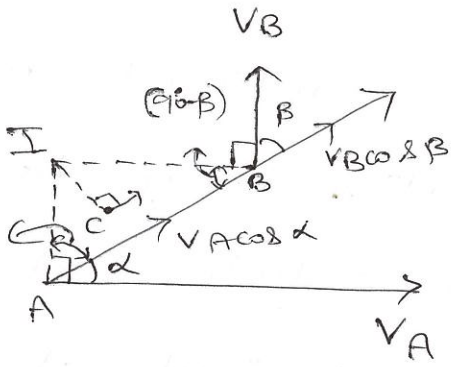
It is equal to no. of pairs.

$$\therefore N = \frac{n(n-1)}{2} \quad (\because n = \text{no. of links}).$$

* Velocity of a point on a link by Instantaneous Centre Method :-

→ consider 2 pts on a rigid link.

→ let $V_A$ & $V_B$ be velocities of A & B, whose directions are given by angles $\alpha$ & $\beta$.




→ $V_A$ is known in magnitude also.

→ Draw ⊥ AI & BI to directions of $V_A$ & $V_B$. Both will intersect at one pt., called I.

→ Now resolve velocities along AB.

$$V_A \cos\alpha = V_B \cos\beta$$

$$\frac{V_A}{V_B} = \frac{\cos\beta}{\cos\alpha}$$

$$\therefore \frac{V_A}{V_B} = \frac{\sin(90^\circ-\beta)}{\sin(90^\circ-\alpha)} \rightarrow (1)$$

→ Now apply lami's theorem to Δ ABI,

$$\frac{AI}{\sin(90^\circ-\beta)} = \frac{BI}{\sin(90^\circ-\alpha)}$$

$$\therefore \frac{AI}{BI} = \frac{\sin(90^\circ-\beta)}{\sin(90^\circ-\alpha)} \rightarrow (2).$$

From (1) & (2), $\frac{V_A}{V_B} = \frac{AI}{BI}$

$$\Rightarrow \frac{V_A}{AI} = \frac{V_B}{BI} = \omega.$$

→ If C is any other pt. on link, then-

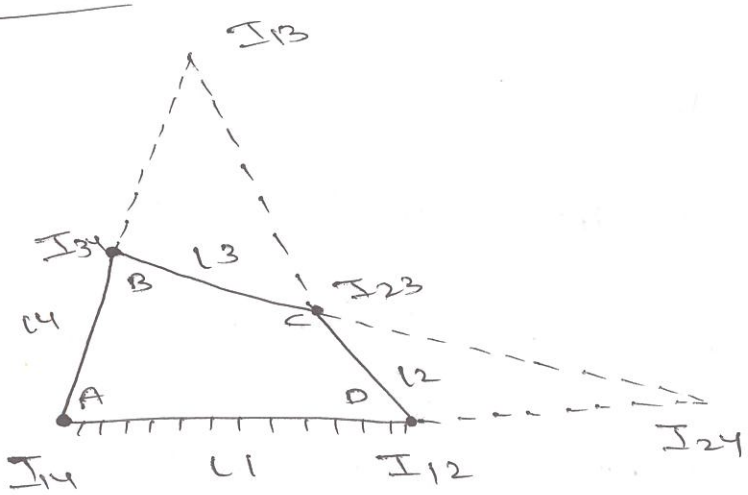
$$\therefore \frac{V_A}{AI} = \frac{V_B}{BI} = \frac{V_C}{CI} = \omega.$$

## * Types of Instantaneous centres :-

→ 1). Fixed IC

2). Permanent IC

3). Neither fixed nor permanent IC.




→ 1st & 2nd Ic are together known as "primary Ic" & 3rd type is known as "secondary Ic".

→ $N = \frac{n(n-1)}{2} = \frac{4(4-1)}{2} = 6.$

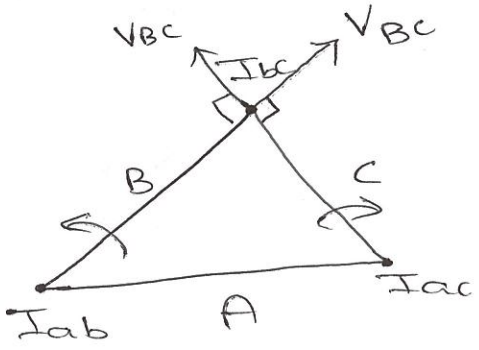
→ $I_{12}, I_{14}$ = FIC; $I_{23}, I_{34}$ = PIC & $I_{13}, I_{24}$ = NFNPIC

# * Aronhold Kennedy (or Three centres in line)

## Theorem :-

→ It states that, if three bodies move relatively to each other, they have 3 Ic's & lie on a st. line

→ $N = \frac{n(n-1)}{2} = \frac{3(3-1)}{2} = 3$




→ Point $I_{bc}$ belongs to both links B & C.

→ Now consider $I_{bc}$ on link B, then its $V_{BC}$ must be ⊥ to line joining $I_{ab}$ & $I_{bc}$.

→ Now again consider $I_{bc}$ on link c, then its $V_{BC}$ must be ⊥ to line joining $I_{ac}$ & $I_{bc}$.

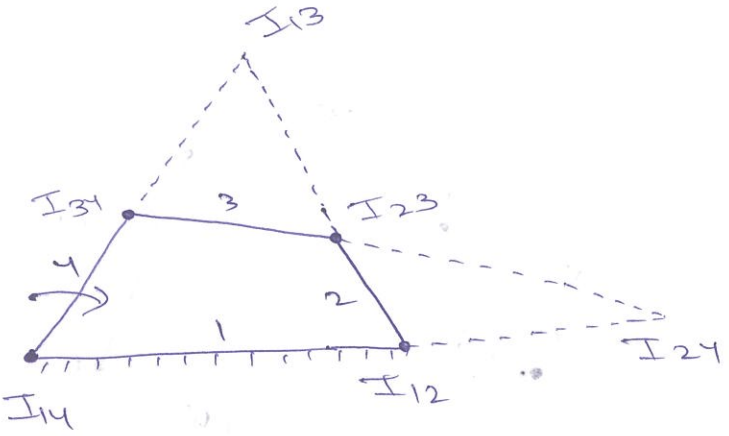
→ But Velocity of pt. $I_{bc}$ cannot be ⊥ to both lines $I_{ab}I_{bc}$ & $I_{ac}I_{bc}$ unless pt. $I_{bc}$ lies on lines joining $I_{ab}$ & $I_{ac}$.

→ Thus 3 IC's must lie on some st. line.

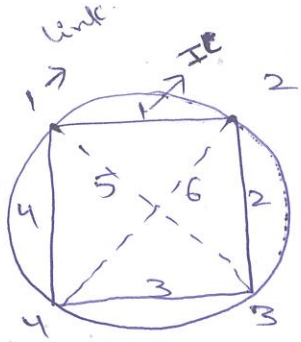
→ Hence proved.

## * Method of locating IC's in a Mechanism

→ $N = \frac{n(n-1)}{2} = \frac{4(4-1)}{2} = 6.$




four bar Mechanism




circle Dgm

→ Book-keeping table :-

| Links | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| IC's (6) | 12<br>13<br>14 | 23<br>24 | 34 | — |

→ located $I_{14}, I_{12}, I_{23}, I_{34}$ by inspection.

→ $I_{13}, I_{24}$ location is done by kennedy theorem. This is done by circle dgm.

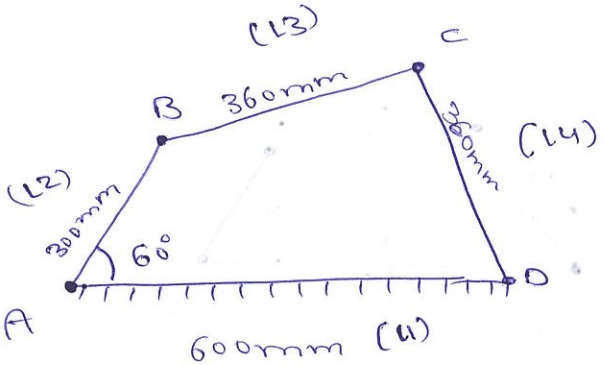
* In a pin jointed four bar mechanism, as shown in fig., AB = 300mm, BC = CD = 360mm, AD = 600mm. The angle BAD = 60°. The crank

AB rotates uniformly at 100rpm. Locate all the instantaneous centres & find angular velocity of link BC.

→ $N_{AB} = 100$ rpm,

$$\omega_{AB} = \frac{2\pi \times 100}{60}$$

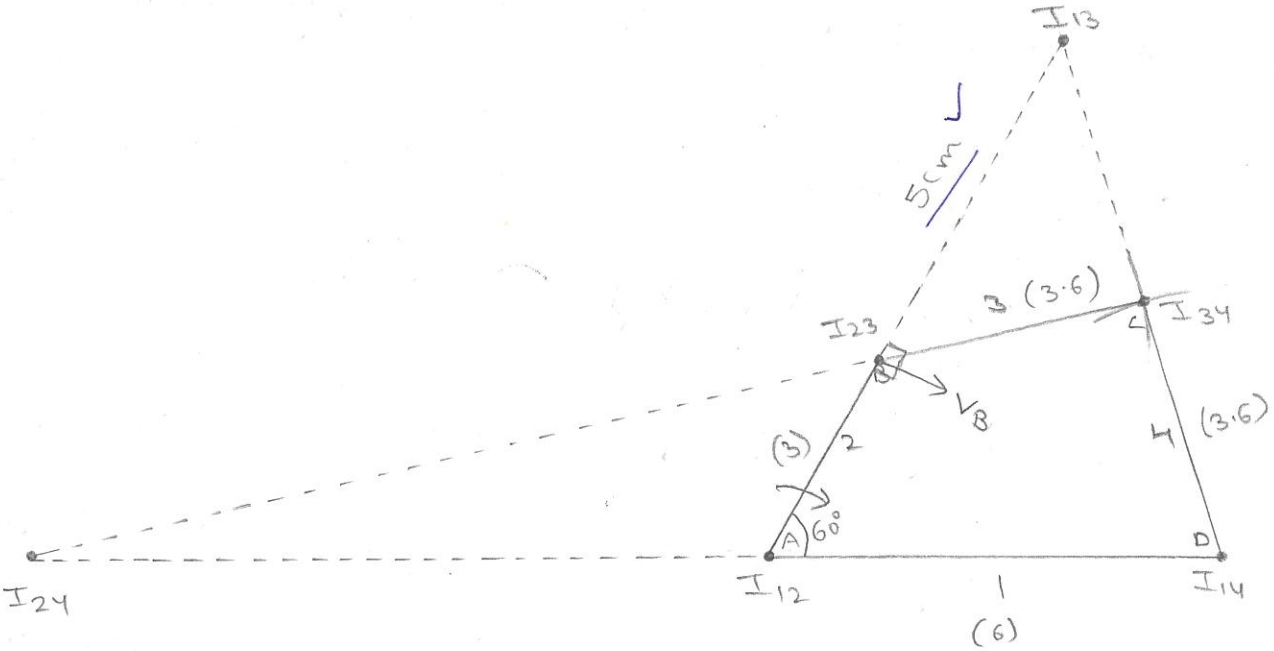
$$= 10.47 \text{ rad/sec}$$




$AB = 300\text{mm} = 0.3\text{m}$.

$$\Rightarrow N = \frac{n(n-1)}{2} = \frac{4(4-1)}{2} = 6.$$

Mechanism Dgm:- Scale:- 100mm = 1cm.



$$\Rightarrow V_{AB} = V_B = \omega_{AB} \times AB$$

$$= 10.47 \times 0.3 = 3.141$$

$$\therefore V_B = 3.141 \text{ m/sec.}$$

→ 'B' is also a pt. on link BC, so velocity of pt B on link BC, $V_B = \omega_{BC} \times I_{13}B$

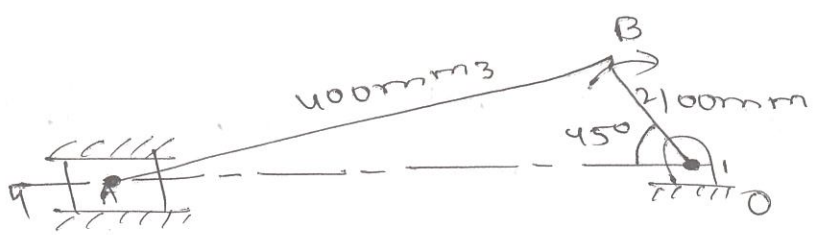
$$\Rightarrow \omega_{BC} = \frac{V_B}{I_{13}B} = \frac{3.141}{0.5}$$

(from measurement, $I_{13}B = 5\text{cm} = 500\text{mm}$

$= 0.5\text{ m}$)

$$\therefore \omega_{BC} = 6.282 \text{ rad/sec.}$$

* Locate all I.C's of slider crank mechanism as shown in fig. The lengths of crank OB & C.Rod AB are 100mm & 400mm res. If the crank rotates clockwise with an angular velocity of 10rad/sec. Find, 1). Velocity of slider A. 2). $\omega_{AB}$.



$\rightarrow$ $\omega_{OB} = 1 \text{ rad/sec}$, $OB = 100\text{mm} = 0.1\text{m}$.

$\Rightarrow V_{OB} = V_B = \omega_{OB} \times OB$

$= 10 \times 0.1 = 1\text{m/sec}$.

$\Rightarrow N = \frac{n(n-1)}{2} = \frac{4(4-1)}{2} = 6$.

Mechanism Dgm :- Scale :- 100mm = 1cm.

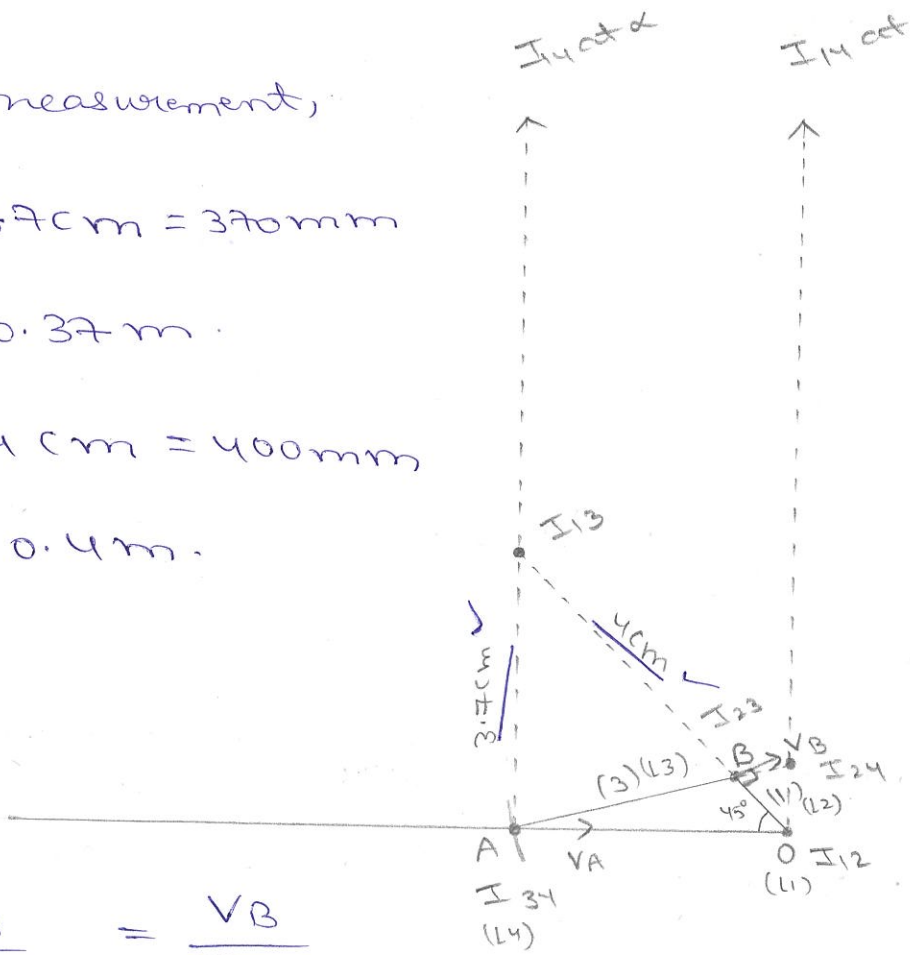
From measurement,

$I_{13A} = 3.7\text{cm} = 370\text{mm}$

$= 0.37\text{m}$.

$I_{13B} = 4\text{cm} = 400\text{mm}$

$= 0.4\text{m}$.




$\Rightarrow \frac{V_A}{I_{13A}} = \frac{V_B}{I_{13B}}$

$\Rightarrow V_B = \frac{V_A}{I_{13A}} \times I_{13B} = \frac{1 \times 0.4}{0.37} = 1.08\text{ m/sec}$.

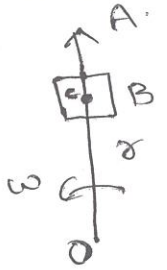
$$\Rightarrow \frac{V_A}{I_{13A}} = \frac{V_B}{I_{13B}} = \omega_{AB}$$

$$\Rightarrow \omega_{AB} = \frac{1.08}{0.4} = 2.7$$

$$\therefore \omega_{AB} = 2.7 \text{ rad/sec}$$

* **coriolis component of Acceleration :- ($a^c$).**

→ When a point on 1 link is sliding along another rotating link, such as in quick return motion mechanism, then $a^c$ must be calculated.




⇒ B = slider, OA = link,
C = coincident point.

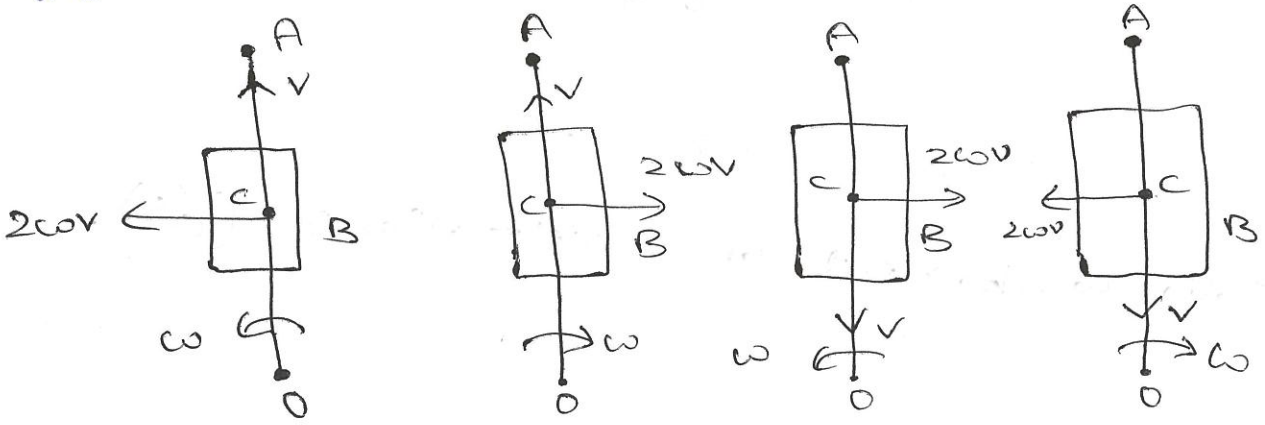
→ Tangential component of acceleration of slider 'B' w.r.t coincident pt 'C' on link 'OA' is known as coriolis component of acc. & is always ⊥ular to the link.

$\therefore a^c$ of B w.r.t C $= a^c_{BC} = a^t_{BC} = 2\omega V_{OA}$

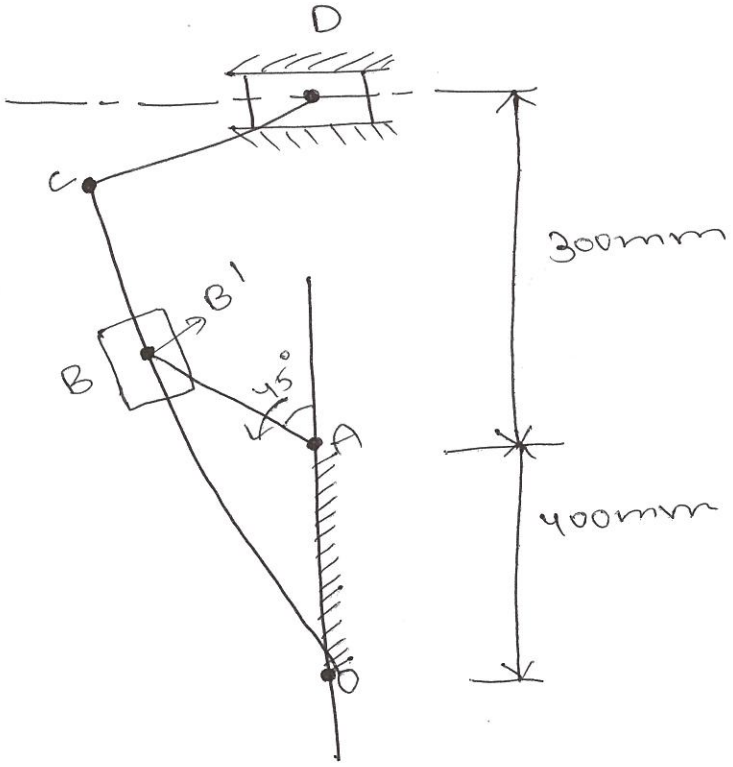
$$\Rightarrow a^c_{BC} = 2\omega_{OA} V_{BC}$$

$$\therefore a^c_{BC} = 2\omega_{OA} V_{BC} \checkmark$$

→ The direction of $a^c$ is obtained by rotating v, at 90°, abt its origin in the same direction as that of ω.




* A mechanism of crank & slotted lever quick return motion is shown in fig. If crank rotates counter clockwise at 120 rpm, crank AB = 150mm, slotted arm OC = 70mm & link CD = 200mm. calculate $a^c$ of 'B wrt B'.



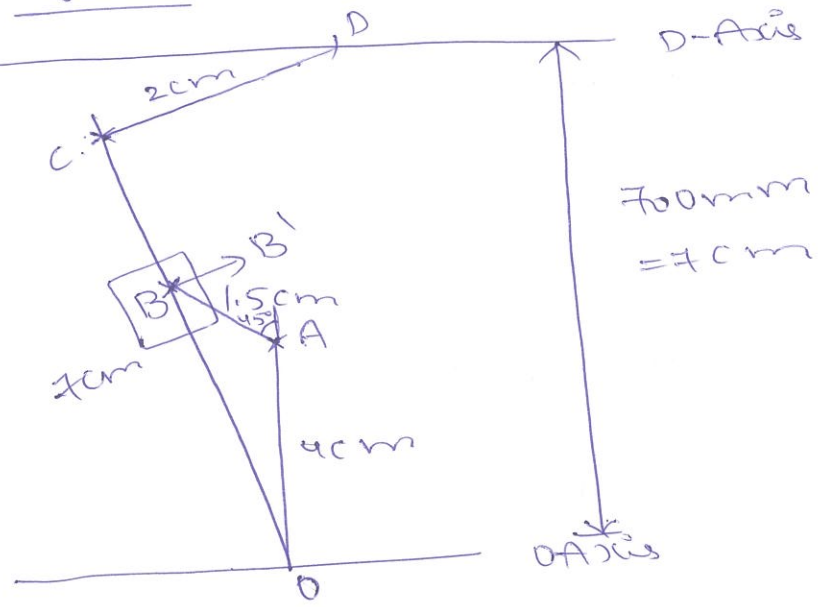
→ $N_{BA} = 120$ rpm $\Rightarrow \omega_{BA} = \frac{2\pi 120}{60} = 12.57$ rad/sec,

$AB = 150\,mm = 0.15\,m$, $OC = 700\,mm = 0.7\,m$,

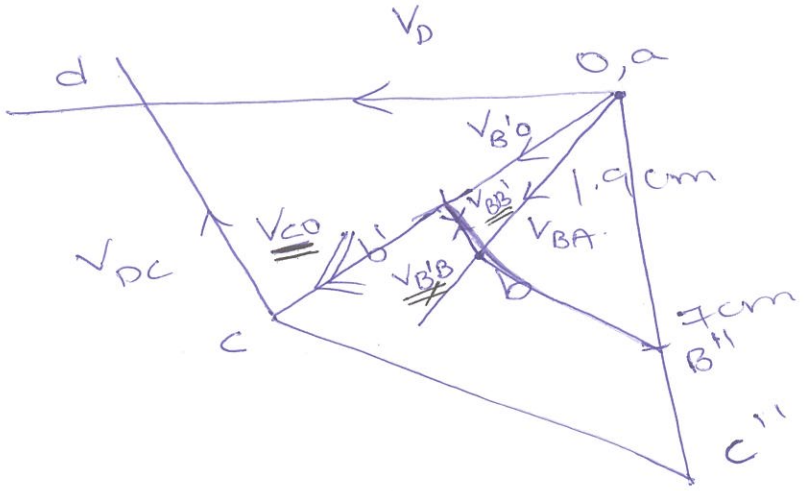
$CD = 200\,mm = 0.2\,m$.

$\Rightarrow V_{BA} = \omega_{BA} \times AB = 12.57 \times 0.15 = 1.9$ m/sec.

Space Dgm :- Scale :- 100mm = 1cm.




Velocity Dgm :- Scale :- 1 m/sec = 1 cm.




From Velocity dgm, calculate $V_{CO}$ & $V_{B'B}$.

From measurement, $V_{BB'} = 1.05 \text{ m/sec}$.

& $V_{co} = 2.15 \text{ m/sec}$.

$$\Rightarrow \omega_{co} = \frac{V_{co}}{oc} = \frac{2.15}{0.7} = 3.07 \text{ rad/sec}.$$

$$\Rightarrow a^c_{BB'} = 2\omega V$$

$$= 2\omega_{co} \times V_{BB'}$$

$$= 2 \times 3.07 \times 1.05$$

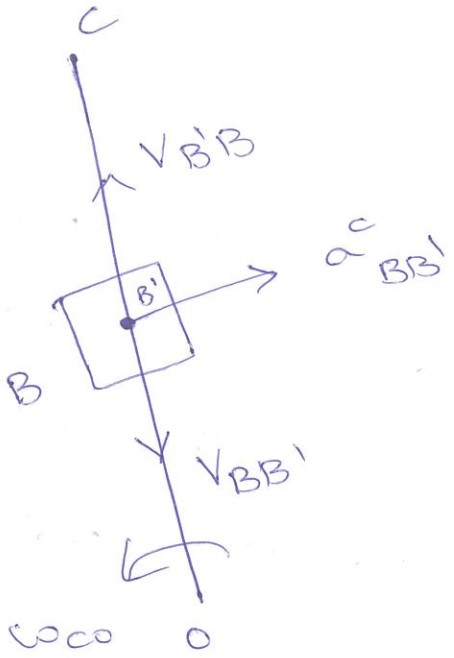
$$= 6.45 \text{ m/s}^2.$$

$$\therefore a^c_{BB'} = 6.45 \text{ m/sec}^2.$$

Here $V_{B'B}$ is in ↑ direction & $V_{BB'}$ is in ↓ direction.

We need Velocity of B wrt B' i.e $V_{BB'}$.

Now, based on that $V_{BB'}$, direction of $a^c_{BB'}$ is obtained.




Direction of coriolis component.

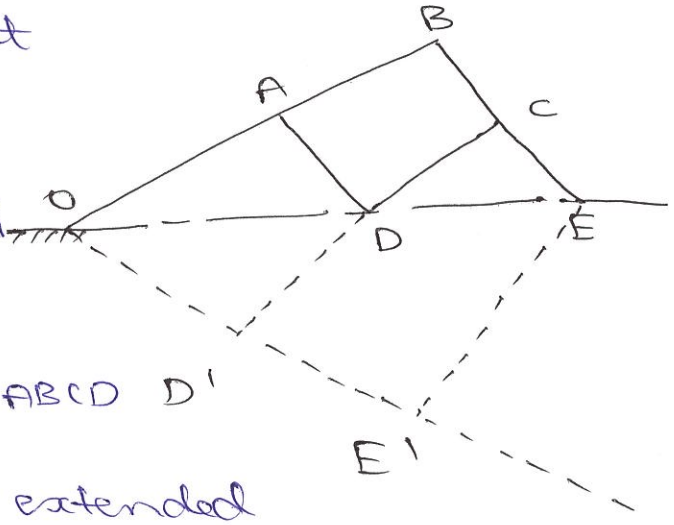
# Straight Line Motion Mechanisms

## * Lower pair :-

When two elements of a pair have a surface contact & relative motion takes place, surface of one element slides over d other, the pair formed is called lower pair.

## * Pantograph :-

→ It is an instrument used to reproduce to an enlarged or a reduced scale.




→ It consists of ||gram ABCD D' & bars BA & BC are extended to O & E, such that $\frac{OA}{OB} = \frac{AD}{BE}$.

→ OAD, OBE are similar, pts O, D, E are in 1st. line & E traces out same path as described by pt. D.

→ OAD & OBE are similar Δles, so $\frac{OD}{DE} = \frac{AD}{BE}$.

→ Let O be fixed, pts. D & E move to some new

positions D' & E', then $\frac{OD}{OE} = \frac{OD'}{OE'}$.

→ If O is fixed, D has rectilinear motion to frame, then E will also trace out a st. line path & vice-versa.

## * St. line Mechanisms :-

→ Mechanisms which permits only relative motion of an oscillatory nature along a st. line are called st. line mechanisms.

→ These are of 2 types →

1). in which only turning pairs are used

2). in which one sliding pair is used.

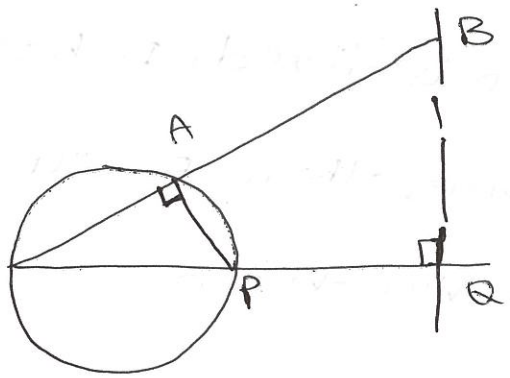
→ Both types may produce exact st. line or approximate st. line motion.

## * Exact st. line motion mechanisms made up of turning pairs :-

→ Let O be a pt. on circumference of a circle of dia. OP.

→ Let OA be any chord & B is a pt. on OA produced

such that, OA × OB = constant.




→ locus of pt. B will be a st. line ⊥ to dia. OP.

This may be proved as follows —

→ Draw BQ ⊥ to OP & join AP.

Δles OAP & OBQ are similar.

$$\Rightarrow \frac{OA}{OP} = \frac{OQ}{OB}$$

$$\Rightarrow OP \times OQ = OA \times OB$$

$$\therefore OQ = \frac{OA \times OB}{OP}.$$

→ As OP is dia, it is constant & if OA × OB is constant, then OQ is also constant.

→ Hence, pt. B moves along st. path BQ which is ⊥ to OP.

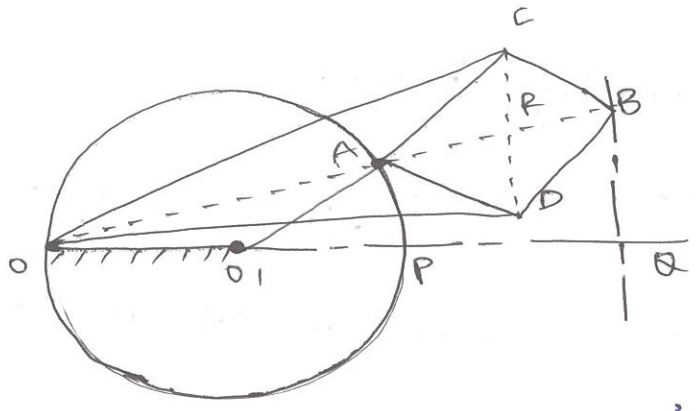
* Types of ESLMM :-

*1). Peaucellier Mechanism :-

→ It consists of fixed link OO, & other st. links

$O_1A$, OC, OD, AD, DB, BC, CA are connected by turning pairs.

→ In fig., $AC = CB = BD = DA$, $OC = OD$, $OO_1 = O_1A$.




→ Prove $OA \times OB$ = constant, it is Lyk —

→ When link $O_1A$ ~~rotates~~, join CD to bisect AB at R, now ORC & BRC are Δles, then —

$$OC^2 = OR^2 + RC^2 \rightarrow (1)$$

$$BC^2 = RB^2 + RC^2 \rightarrow (2)$$

$$\Rightarrow (1) - (2) \Rightarrow OC^2 - BC^2 = OR^2 - RB^2$$

$$= (OR + RB)(OR - RB)$$

$$= OB \times OA$$

→ Since OC & BC are of constant length, $OB \times OA$ is also constant.

→ Hence, pt. B traces a st. line path ⊥ to dia. OP.

+2). Hart's mechanism:-

→ It requires only 6 links. It consists of fixed link $OO_1$ & other st. links $O_1A$, FC, CD, DE & EF are connected by turning pairs.

→ In this, FC = DE, CD = EF, OA & OB || FD & CE.

→ Prove OA × OB = constant, it is lyk —

→ from Δles CFE, OFB ⇒

$$\frac{CE}{FC} = \frac{OB}{OF} \Rightarrow OB = \frac{CE \times OF}{FC} \rightarrow (1).$$

from Δles FCD, OCA ⇒

$$\frac{FD}{Fc} = \frac{OA}{OC} \Rightarrow OA = \frac{FD \times OC}{Fc} \rightarrow (2).$$

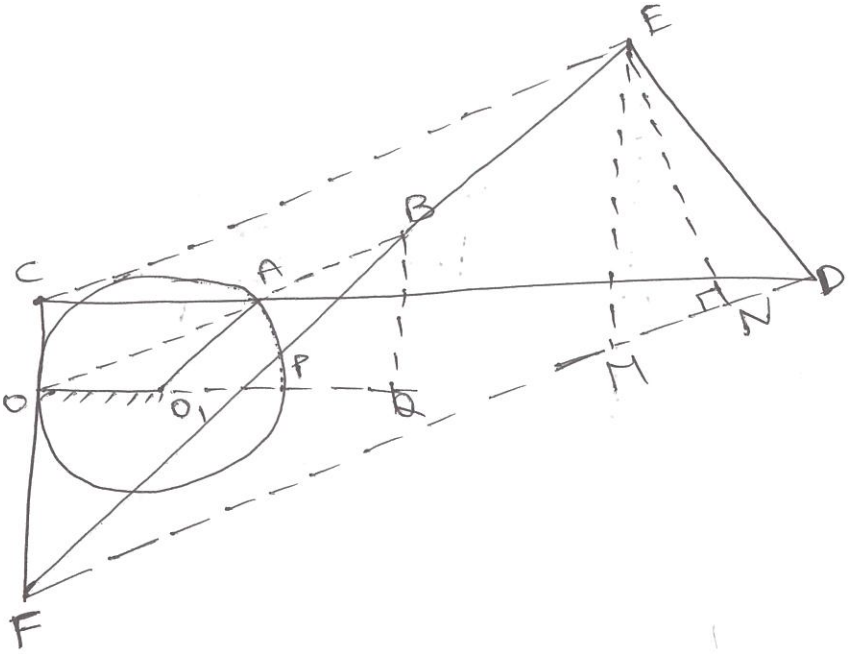
$$\Rightarrow (1) \times (2) \Rightarrow \frac{FD \times OC}{FC} \times \frac{CE \times OF}{FC}$$

$$= FD \times CE \times \frac{OC \times OF}{FC^2}$$

∴ OC, OF, FC are fixed, then —

$$\Rightarrow OA \times OB = FD \times CE \times \text{Constant} \rightarrow (3)$$

→ Now from pt. E, draw EM || to CF & EN ⊥ to FD.




$\Rightarrow$ $FD \times CE = FD \times FM$ $(\because CE = FM)$.

$= (FN + ND)(FN - MN)$

$= FN^2 - ND^2$ $(\because MN = ND)$.

$= (FE^2 - NE^2) - (ED^2 - NE^2)$.

$\rightarrow$ (4).

(from $\Delta$les FEN & EDN).

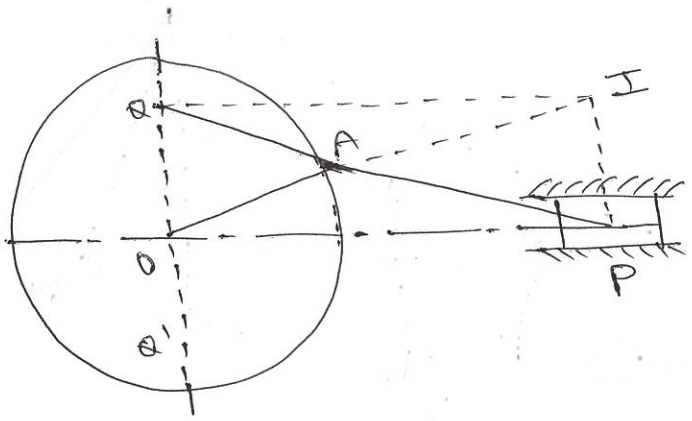
$\Rightarrow FE^2 - ED^2 = \text{constant}$.

from (3) & (4), $OA \times OB = \text{constant}$.

$\rightarrow$ Hence pt. B will trace a st. line $\perp$ to dia. OP.

* Exact st. line Motion consisting of one sliding pair - Scott Russell's Mechanism :-

$\rightarrow$ It is same as reciprocating engine mechanism.




→ OA = AP = AQ & locate I such that IP ⊥ OP.

→ Join IQ, here OPIQ is rectangle & IQ is ⊥ to OQ, so Q moves along vertical line OQ for all positions of OP.

→ Hence Q traces st. line OQ'.

## * Approximate st. line Motion Mechanisms:-

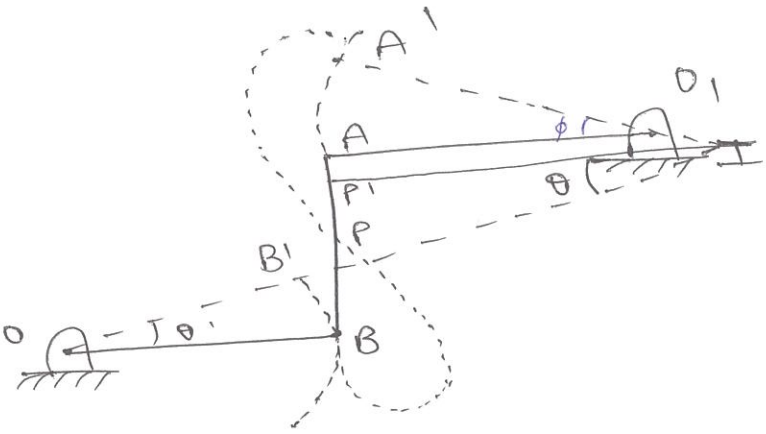
These are modifications of four-bar chain mechanisms.

## * Types of ASLMM:-

### *1). Watt's Mechanism:-

→ It is a crossed four bar chain mechanism.

→ In fig. OBAO, is a crossed 4bar chain in which O & O, are fixed.




→ Links OB & $O_1A$ are || & rod AB is ⊥ to $O_1A$ & OB.

→ The tracing pt. P traces out an approximate st. line, if $\frac{PB}{PA} = \frac{O_1A}{OB}$.

→ Proved as follows →

→ Let $OB'A'O_1$ be new position of the mechanism after links OB & $O_1A$ are displaced through an angle θ & φ.

$\Rightarrow$ ~~Arc~~ $BB' = AA'$

$\Rightarrow OB \times \theta = O_1A \times \phi$

$\therefore \frac{OB}{O_1A} = \frac{\phi}{\theta} \rightarrow (1)$

And also,

from $\Delta A'P'I$, $\phi = \frac{A'P'}{IP'} \Rightarrow A'P' = IP' \times \phi$.

from $\Delta B'P'I$, $\theta = \frac{B'P'}{IP'} \Rightarrow B'P' = IP' \times \theta$.

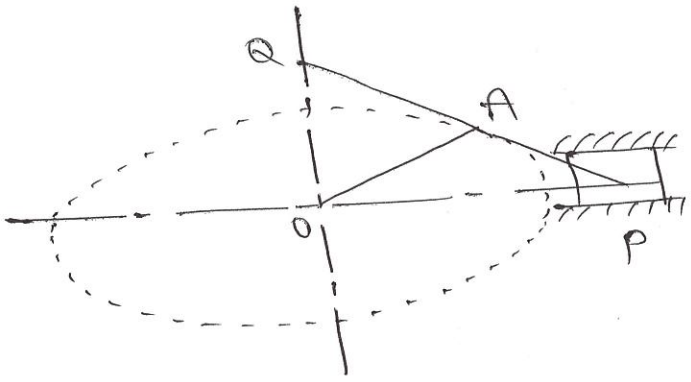
$$\therefore \frac{A'P'}{B'P'} = \frac{\phi}{\theta} \rightarrow (2).$$

from (1) & (2), $\frac{OB}{OA} = \frac{A'P'}{B'P'} = \frac{AP}{BP}$

$$\therefore \frac{OA}{OB} = \frac{PB}{PA}.$$

*2). Modified Scott-Russel Mechanism:-

→ It is similar to Scott-Russel mechanism.

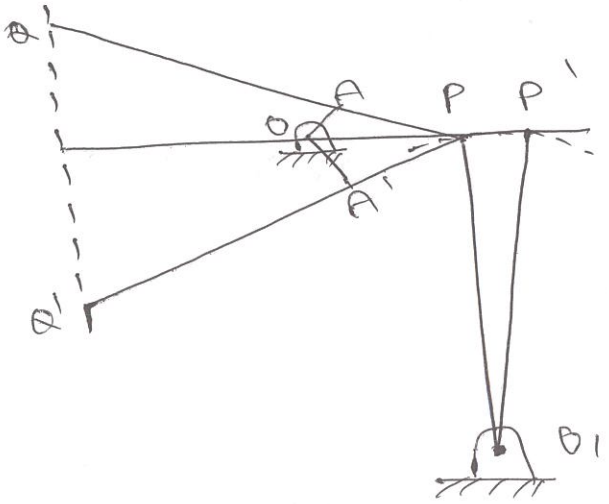



→ Here AP ≠ AQ & it forms an elliptical trammel, so that any pt. A on PQ traces an ellipse with semi-major axis AQ & semi minor axis AP.

→ If pt. A moves in a circle, then pt. Q move along appro. st line, length OA must be equal to $\frac{(AP)^2}{AQ}$.

## *3). Grasshopper mechanism :-

→ This is modification of modified Scott-Russel's mechanism, with difference that pt P does not slide along st line, but moves in a circular arc with centre O.

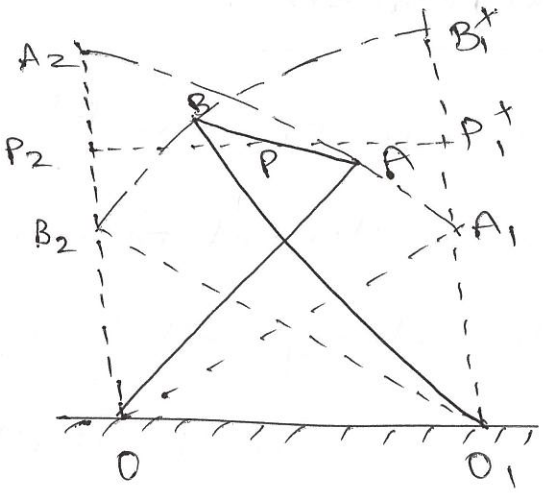



→ Pt. Q on extension of link PA traces out an appro. st. path QQ', if lengths are such that $OA = \frac{(AP)^2}{AQ}$.

## * 4). Tchebicheff's Mechanism :-

→ In this OA = O,B & pt P which is mid-pt. of AB traces out an app. st. line ⊥ to OO,

→ It will traces out a st. path if lengths of links are in proportions AB : OO, : OA = 1 : 2 : 2.5.

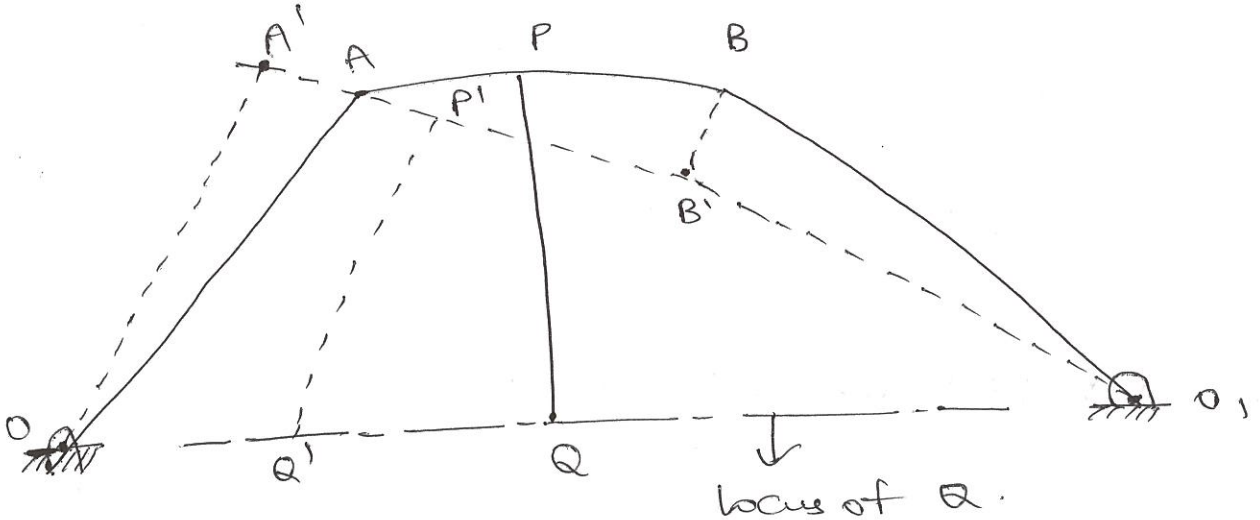



## * 5). Roberts mechanism :-

→ It is also four bar chain in its mean position has form of a trapezium.

→ OA = O1B & O1O is fixed.

→ Q pt. will trace out an appr. st. line.




## * Steering Gear Mechanism :-

→ It is used for changing the direction of two or more of the wheel axes with ref. to chasis,

so as to move automobile in any desired path.

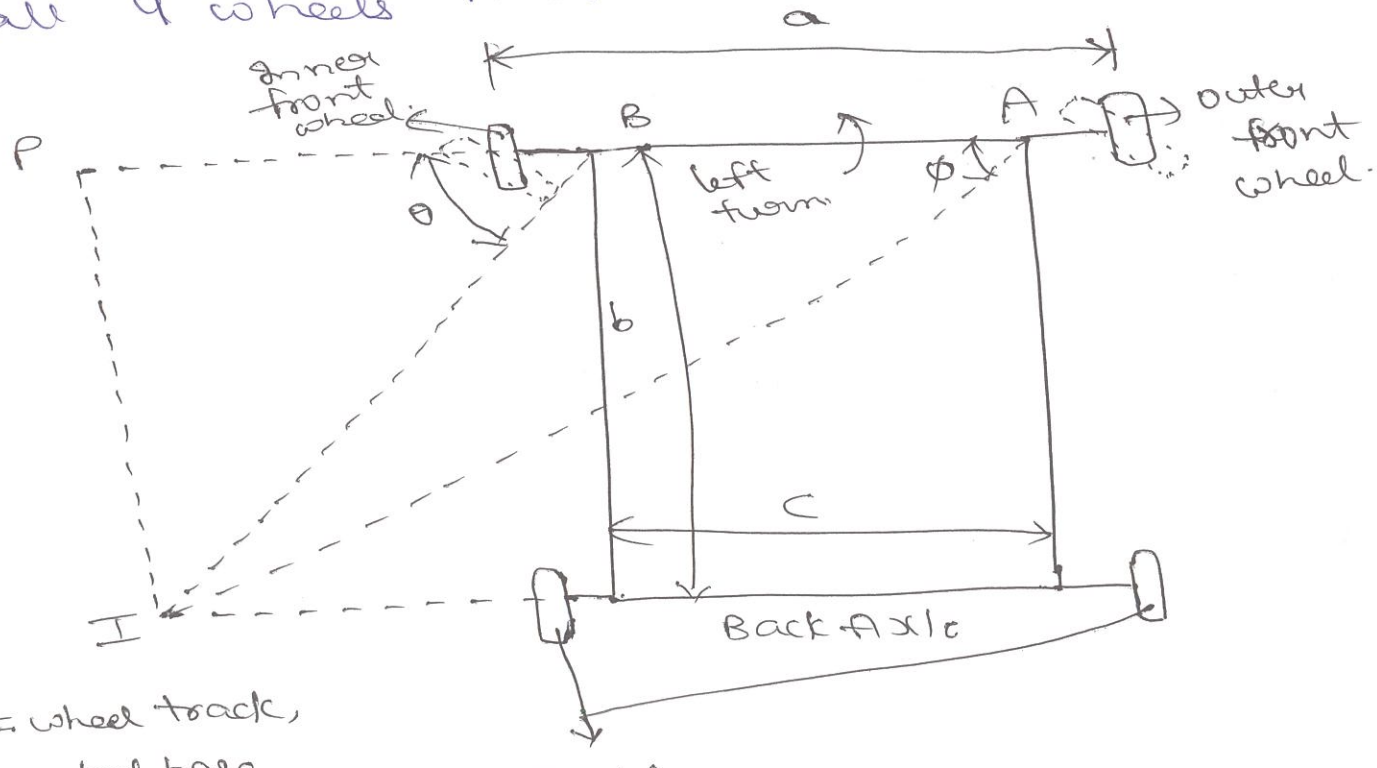
→ Steering is done by means of front wheels only.

→ When vehicle takes a turn, front wheels along with respective axles turn abt respective pivoted pts.

→ Back wheels remain st. & do not turn.

→ In order to avoid skidding, the two front wheels must turn about same 'I' which lies on axis of back wheels.

→ Thus, condition for correct steering is that all 4 wheels must turn abt. same I.




{ a = wheel track,
b = wheel base,
c = Dis. b/w pivots A & B }.

from $\Delta IBP$, $\cot\theta = \frac{BP}{IP}$.

from $\Delta IAP$, $\cot\phi = \frac{AP}{IP}$.

$$= \frac{AB+BP}{IP} = \frac{AB}{IP} + \frac{BP}{IP}.$$

$$= \frac{c}{b} + \cot\theta.$$

$$\therefore \cot\phi - \cot\theta = \frac{c}{b}.$$

→ This is the fundamental eqn for correct steering.

→ If this condition is satisfied, there will be no skidding of wheels, when vehicle takes a turn.

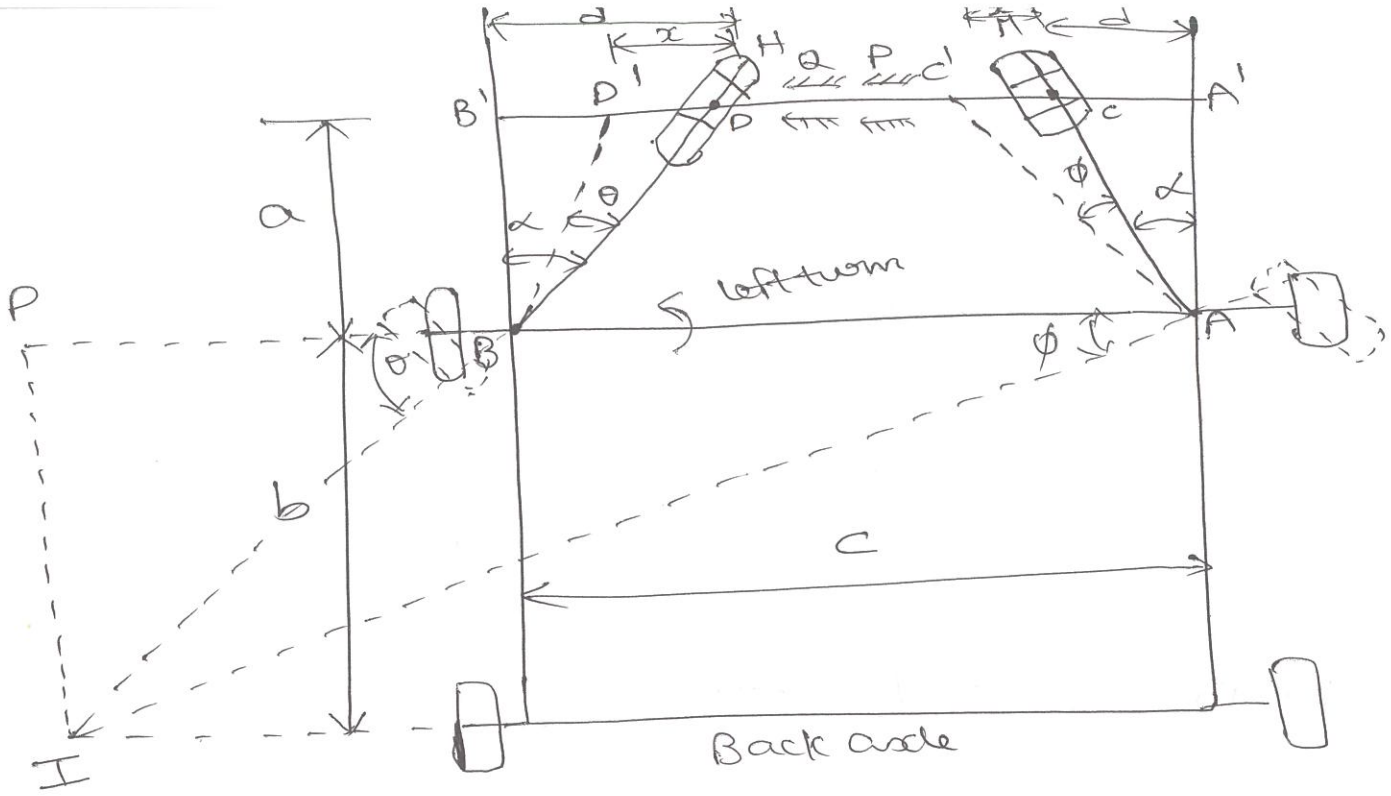
## * Davis steering gear :-

→ It is an exact steering gear mechanism.

→ Two slotted links AM & BH are attached to front wheel axle.

→ The rod CD is constrained to move by sliding members at P and Q.

→ C'D' shows position of CD for turning to left.




from $\Delta AA'c'$, $\tan(\alpha+\phi) = \frac{A'c'}{AA'} = \frac{d+x}{a} \rightarrow (1)$

from $\Delta AA'C$, $\tan\alpha = \frac{A'C}{AA'} = \frac{d}{a} \rightarrow (2)$

from $\Delta BB'D$, $\tan(\alpha-\theta) = \frac{B'D'}{BB'} = \frac{d-x}{a} \rightarrow (3)$

from (1), $\tan(\alpha+\phi) = \frac{(\tan\alpha+\tan\phi)}{1-\tan\alpha\tan\phi}$

$$\Rightarrow \frac{d+x}{a} = \frac{d/a+\tan\phi}{1-\frac{d}{a}\tan\phi} = \frac{d+a\tan\phi}{-d\tan\phi+a}$$

$$\Rightarrow (d+x)(a-d\tan\phi) = a(d+a\tan\phi)$$

$$\Rightarrow ad - d^2\tan\phi + ax - dx\tan\phi$$

$$= ad + a^2\tan\phi$$

$$\Rightarrow \tan\phi\,(a^2+d^2+dx) = ax$$

$$\Rightarrow \tan\phi = \frac{ax}{a^2+d^2+dx} \quad \checkmark$$

from ③, similarly, $\tan(\alpha-\theta) = \frac{d-x}{a}$

$$\Rightarrow \tan\theta = \frac{ax}{a^2+d^2-dx} \quad \checkmark$$

We know, $\cot\phi - \cot\theta = \frac{c}{b}$

$$\Rightarrow \frac{1}{\tan\phi} - \frac{1}{\tan\theta} = \frac{c}{b}.$$

$$\Rightarrow \frac{a^2+d^2+dx}{ax} - \frac{a^2+d^2-dx}{ax} = \frac{c}{b}$$

$$\Rightarrow \frac{2dx}{ax} = \frac{c}{b}$$

$$\Rightarrow \frac{2d}{a} = \frac{c}{b}$$

$$\Rightarrow 2\times\tan\alpha = \frac{c}{b} \quad (\because \text{from } ②).$$

$$\Rightarrow \tan\alpha = \frac{c}{2b}.$$

$$\therefore \tan\alpha = \frac{c}{2b}.$$

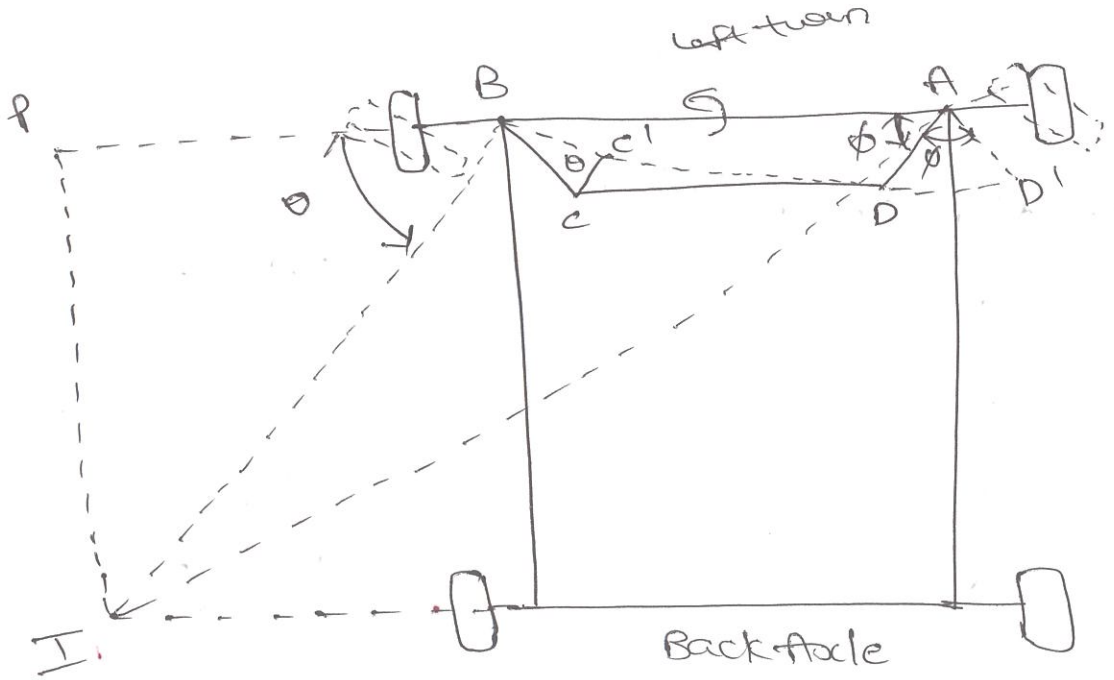
* Ackerman steering Gear :-

→ It is much simpler than davis gear.

→ Differences b/w two types are :-

1). Whole mechanism of A.S.G is on back of front wheels & in D.S.G, it is front of the wheels.

2). A.S.G consists of turning pairs & D.S.G consists of sliding member.




* In a Davis steering gear, distance b/w the pivots of front axle is 1.2 mts & wheel base is 2.7 mts. Find inclination of track arm to longitudinal axis of car, when it is moving along

a st. path.

→ $c = 1.2\,m,\ b = 2.7\,m,\ \alpha = ?$

$$\Rightarrow \tan\alpha = \frac{c}{2b} = \frac{1.2}{2 \times 2.7} = 0.222$$

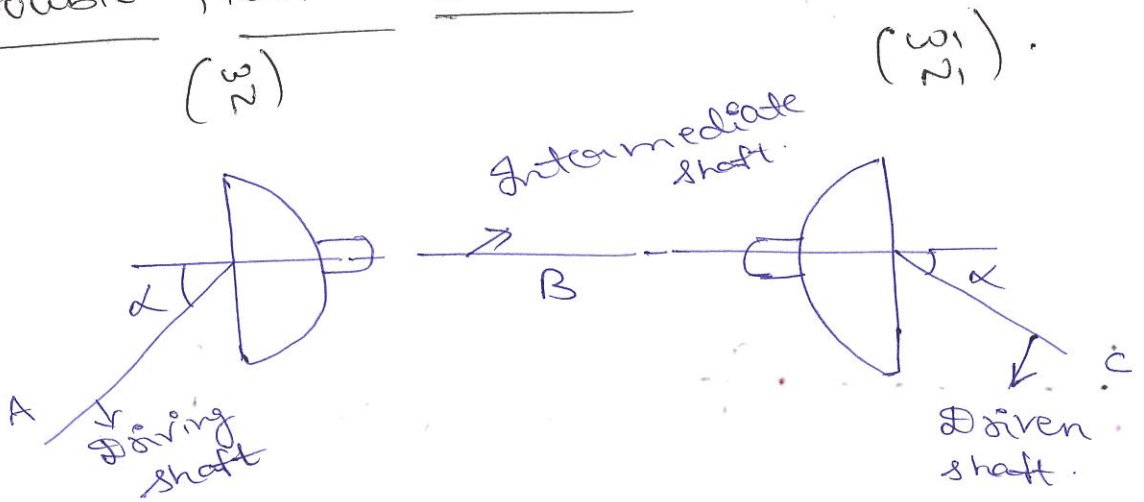
$$\therefore \alpha = 12.5^\circ.$$

* <u>universal (or) Hooke's joint</u> :-

→ It is used to connect two shafts, which are intersecting at a small angle.

→ It is used to transmit power from gear box to back axle of automobiles.

* <u>Double Hooke's joint</u> :-




→ To have constant velocity ratio of the driving & driven shafts, intermediate shaft with a hooke's jt at each end is used.

→ This type of jt. is called Double Hooke's joint.

## * Formulae :-

1). Ratio of shafts velocities, $\frac{\omega_1}{\omega_2} = \frac{N_1}{N}$

$$= \frac{\cos\alpha}{1-\cos^2\theta\sin^2\alpha}.$$

2). If $\cos^2\theta = 1$, $\omega_1$ will be max,

$$\Rightarrow \omega_{1(max)} = \frac{\omega}{\cos\alpha}$$

$$\Rightarrow N_{1(max)} = \frac{N}{\cos\alpha}.$$

3). If $\cos^2\theta = 0$, $\omega_1$ will be mini,

$$\Rightarrow \omega_{1(mini)} = \omega\cos\alpha$$

$$\Rightarrow N_{1(mini)} = N\cos\alpha.$$

4). Condition for equal speeds, $\tan\theta = \pm\sqrt{\cos\alpha}$

5). Angular acc., $\frac{d\omega_1}{dt} = \frac{-\omega^2\cos\alpha \times \sin 2\theta \times \sin^2\alpha}{(1-\cos^2\theta\sin^2\alpha)^2}$

6). Max. Angular acc., $\cos 2\theta = \frac{\sin^2\alpha\,(2-\cos^2 2\theta)}{2-\sin^2\alpha}$

7). If $\alpha < 30°$, $\cos 2\theta = \frac{2\sin^2\alpha}{2-\sin^2\alpha}$.

8).(9) Max. fluctuation of speed $= \omega\alpha^2(\omega) \omega\left(\frac{1-\cos^2\alpha}{\cos\alpha}\right)$

* Two shafts with an included angle of 160° are connected by Hooke's jt. The driving shaft runs at a uniform speed of 1500 rpm. The driven shaft carries a flywheel of mass 12kg & 100mm radius of gyration. Find max. Angular acc. of driven shaft & max. torque required.

→ $\alpha = 180° - 160° = 20°$, $N = 1500$ rpm,

$m = 12$ kg, $k = 100mm = 0.1m$.

$\Rightarrow \omega = \frac{2\pi N}{60} = \frac{2 \times \pi \times 1500}{60} = 157$ rad/sec.

$\Rightarrow I = mk^2 = 12 (0.1)^2 = 0.12$ kg-m$^2$.

$\Rightarrow \cos 2\theta = \frac{2 \sin^2 \alpha}{2 - \sin^2 \alpha} = \frac{2 \sin^2 20°}{2 - \sin^2 20°}$

$\therefore \theta = 41.45°$ $(\because \theta < 30°)$.

$\Rightarrow \frac{d\omega_1}{dt} = \frac{\omega^2 \cos\alpha \sin 2\theta \sin^2 \alpha}{(1 - \cos^2\theta \sin^2\alpha)^2}$

$= \frac{(157)^2 \cos 20° \times \sin 2 \times 41.45° \times \sin^2 20°}{(1 - \cos^2(41.45°) \sin^2 20°)^2}$

$\therefore \frac{d\omega_1}{dt} = 3090$ rad/s$^2$.

$\therefore T = I \times \frac{d\omega_1}{dt} = 0.12 \times 3090$

$= 371\ N\text{-}m.$

* The angle b/w axes of two shafts connected by Hooke's jt. is 18°. Determine angle turned through by driving shaft when velocity ratio is max. & unity.

→ $\alpha = 18°, \quad \frac{\omega_1}{\omega} = \frac{\cos\alpha}{1 - \cos^2\theta \sin^2\alpha}$

1). If $\cos^2\theta = 1$, $\omega_1$ will be max, so

$\Rightarrow \cos^2\theta = 1 \quad \Rightarrow \theta = \cos^{-1}(\sqrt{1}) = 0°.$

$\Rightarrow \theta = 0° \text{ (or) } 180°$ ✓

2). $\frac{\omega_1}{\omega} = 1 \Rightarrow 1 - \cos^2\theta \sin^2\alpha = \cos\alpha.$

$\cos^2\theta = \frac{1 - \cos\alpha}{\sin^2\alpha} = \frac{1 - \cos\alpha}{1 - \cos^2\alpha}$

$\cos^2\theta = \frac{1 - \cos\alpha}{(1 - \cos\alpha)(1 + \cos\alpha)} = \frac{1}{1 + \cos\alpha}.$

$\cos^2\theta = \frac{1}{1 + \cos 18°} = 0.512 \Rightarrow \theta = \cos^{-1}(\sqrt{0.512}).$

$\therefore \theta = 44.3° \text{ (or) } 135.7°$ ✓

* Two shafts are connected by a Hooke's joint. The driving shaft revolves uniformly at 500rpm. If total permissible variation in speed of the driven shaft is not to exceed ±6% of the mean speed, find greatest permissible angle b/w the centre lines of the shafts.

→ $N = 500 rpm$, $\omega = \frac{2\pi \times 500}{60} = 52.4 rad/sec.$

$\Rightarrow q = 12\%$ of mean speed$(\omega)$ $\Rightarrow q = 0.12\omega.$

$$\Rightarrow q = \omega\left(\frac{1-\cos^2\alpha}{\cos\alpha}\right)$$

$$\Rightarrow \omega\left(\frac{1-\cos^2\alpha}{\cos\alpha}\right) = 0.12\omega$$

$$\Rightarrow 1-\cos^2\alpha = 0.12\cos\alpha.$$

$$\Rightarrow \cos^2\alpha + 0.12\cos\alpha - 1 = 0$$

We know, $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$$\Rightarrow \cos\alpha = \frac{-0.12 \pm \sqrt{(0.12)^2+4}}{2} = 0.9418$$

(Take + sign).

$\therefore \alpha = 19.64^\circ$ ✓

* Two shafts are connected by a universal joint. The driving shaft rotates at a uniform speed of 1200 rpm. Determine greatest permissible angle b/w shaft axes so that total fluctuation of speed does not exceed 100 rpm. Also calculate max. & mini speeds of driven shaft.

→ $N = 1200$ rpm, $q = 100$ rpm.

$$\Rightarrow q = N\left(\frac{1-\cos^2\alpha}{\cos\alpha}\right)$$

$$\Rightarrow 100 = 1200\left(\frac{1-\cos^2\alpha}{\cos\alpha}\right).$$

$$\Rightarrow \frac{1-\cos^2\alpha}{\cos\alpha} = \frac{100}{1200} = 0.083$$

$$\Rightarrow \cos^2\alpha + 0.083\cos\alpha - 1 = 0.$$

$$\Rightarrow \cos\alpha = \frac{-0.083 \pm \sqrt{(0.083)^2 + 4}}{2} = 0.95$$

(Take + sign)

$$\therefore \alpha = 16.4^\circ \checkmark$$

$$\Rightarrow N_{1(max)} = \frac{N}{\cos\alpha} = \frac{1200}{\cos 16.4^\circ} = 1251 \text{ rpm} \checkmark$$

$$\Rightarrow N_{1(mini)} = N\cos\alpha = 1200\cos 16.4^\circ = 1151 \text{ rpm} \checkmark$$

* The driving shaft of a Hooke's joint runs at a uniform speed of 240 rpm. & angle $\alpha$ b/w the shafts is 20°. The driven shaft with attached masses has a mass of 55 kg at a radius of gyration of 150 mm.

1). If a steady torque of 200 N-m resists rotation of driven shaft, find torque required at driving shaft, when $\theta = 45°$.

2). At what value of '$\alpha$' will the total fluctuation of speed of driven shaft be limited to 24 rpm?

→ $N = 240$ rpm $\Rightarrow \omega = \frac{2\pi \times 240}{60} = 25.14$ rad/sec,

$\alpha = 20°$, $m = 55$ kg, $k = 150$ mm $= 0.15$ m,

Torque which resists driven shaft $(T_1) = 200$ N-m

1). Torque which accelerates driven shaft $(T_2) = ?$

let $T'$ be torque req at driving shaft.

let $T$ be total torque req. at driven shaft.

$\Rightarrow I = mk^2 = 55 (0.15)^2 = 1.24$ kg-m$^2$.

$$\Rightarrow \frac{d\omega_1}{dt} = \frac{-\omega^2 \cos\alpha \sin 2\theta \sin^2\alpha}{(1-\cos^2\theta \sin^2\alpha)^2}$$

$$= \frac{-(25.14)^2 \cos 20^\circ \times \sin 90^\circ \times \sin^2 20^\circ}{(1-\cos^2 45^\circ \sin^2 20^\circ)^2}$$

$$\therefore \frac{d\omega_1}{dt} = -78.4 \text{ rad/sec}^2 \checkmark$$

$$\Rightarrow T_2 = I \times \frac{d\omega_1}{dt} = 1.24 \times -78.4 = -97.2 \text{ N-m} \checkmark$$

$$\therefore T = T_1 + T_2 = 1.24\ 200 - 97.2 = 102.8 \text{ N-m}$$

We know, $T'\omega = T\omega_1$

$$T' = \frac{T\omega_1}{\omega} = \frac{T\cos\alpha}{1-\cos^2\theta \sin^2\alpha}$$

$$\Rightarrow T' = \frac{102.8 \cos 20^\circ}{1-\cos^2 45^\circ \sin^2 20^\circ} = 102.6 \text{ N-m} \checkmark$$

2). $$q = N\left(\frac{1-\cos^2\alpha}{\cos\alpha}\right)$$

$$\Rightarrow 24 = 240\left(\frac{1-\cos^2\alpha}{\cos\alpha}\right).$$

$$\Rightarrow \frac{1-\cos^2\alpha}{\cos\alpha} = \frac{24}{240} = 0.1$$

$$\Rightarrow \cos^2\alpha + 0.1\cos\alpha - 1 = 0$$

$$\cos\alpha = \frac{-0.1 \mp \sqrt{(0.1)^2 - 4}}{2} = 0.95 \quad (\text{Take + sign})$$

$$\therefore \alpha = 18.2^\circ.$$

* A double universal joint is used to connect two shafts in the same plane. The intermediate shaft is inclined at an angle of 20° to the driving shaft as well as driven shaft. Find max. & mini. speed of intermediate shaft & driven shaft if driving shaft has a constant speed of 500rpm.

→ $\alpha = 20^\circ$, $N_A = 500$ rpm

Drivings — I.S — Drivens

A — B — C

$$\Rightarrow N_{B(max)} = \frac{N_A}{\cos\alpha}$$

$$= \frac{500}{\cos 20^\circ} = 532.1 \text{ rpm} \checkmark$$

$$\Rightarrow N_{B(mini)} = N_A \cos\alpha = 500\cos 20^\circ = 469.85 \text{ rpm} \checkmark$$

$$\Rightarrow N_{C(max)} = \frac{N_{B(max)}}{\cos\alpha} = \frac{532.1}{\cos 20^\circ} = 566.25 \text{ rpm} \checkmark$$

$$\therefore N_{C(mini)} = N_{B(mini)} \times \cos\alpha$$

$$= 500 \times \cos 20$$

$$= 469.85 \times \cos 20^\circ$$

$$= 441.5 \text{ rpm} \sim$$

$\therefore N_{B(max)} = 532.1$, $N_{B(mini)} = 469.85$,

$N_{C(max)} = 566.25$, $N_{C(mini)} = 441.5$.



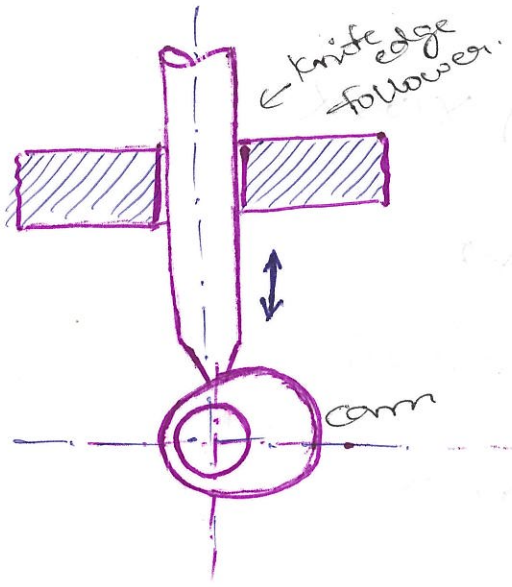
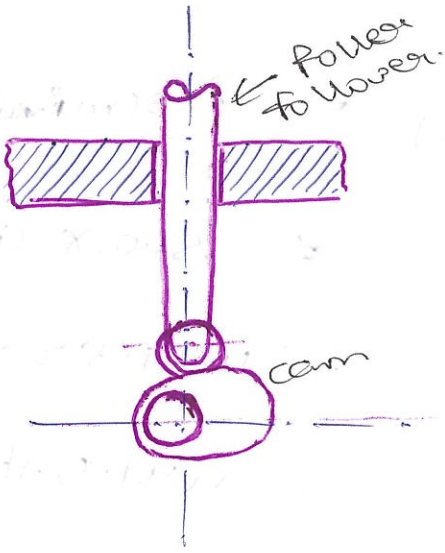
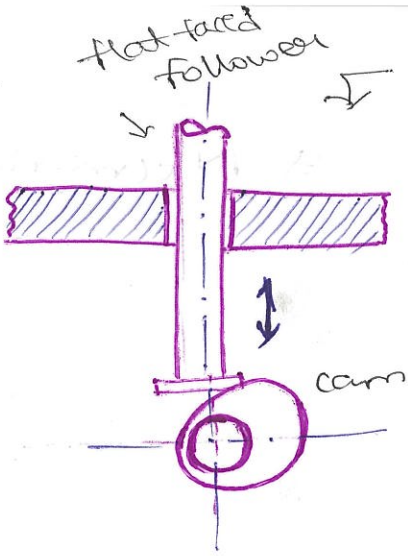


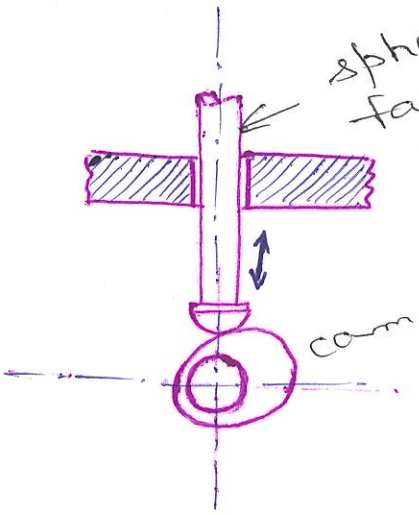

fig. 20.1(a)



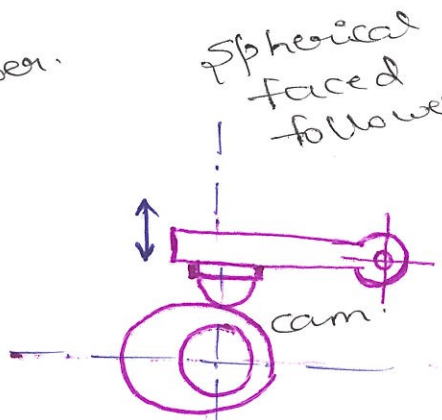

20.1(b)



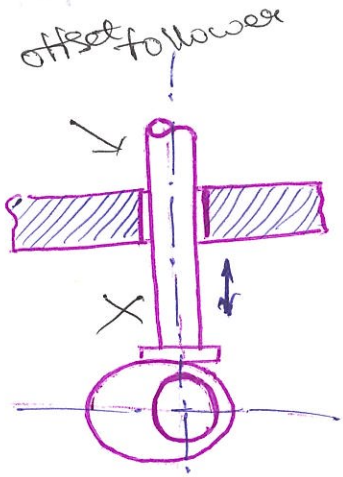

20.1(c)



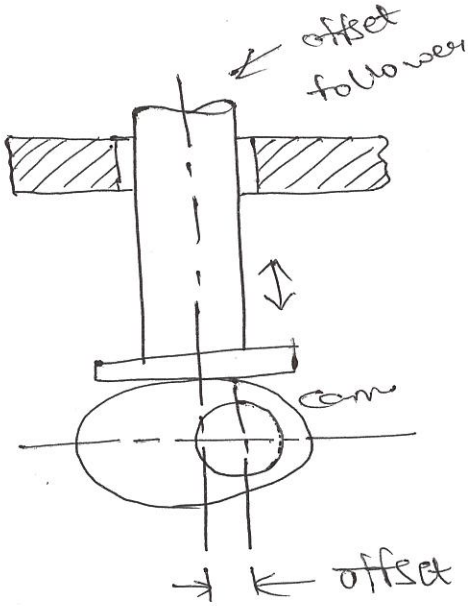

20.1(d)




20.1(e)




20.1(f).




Stud

# CAMS

* cam :-

→ A cam is a rotating m/c element which gives reciprocating (or) oscillating motion to another element known as follower.

→ They are widely used for operating the inlet & exhaust valves of IC engines, paper cutting m/c's, spinning n weaving textile m/c's, feed mechanism of automatic lathes, etc.

* classification of followers :-

* 1). Acc. to surface in contact :-

(a). knife edge follower :-

When contacting end of the follower has a sharp knife edge, it is called a knife edge follower. Sliding motion takes place b/w contacting surfaces.

(b). Roller follower :-

When contacting end of follower is a roller, it is called a roller follower. Rolling motion takes place b/w contacting surfaces.

(c). Flat faced (or) mushroom follower:-

When contacting end of follower is flat face, it is called flat faced follower. When flat faced follower is circular, it is then called a mushroom follower.

(d). Spherical faced follower:-

When contacting end of follower is of spherical shape, it is called spherical faced follower. To minimise high surface stresses which are produced by flat faced, we will use spherical faced follower.

*2). According to the motion of the follower:-

(a). Reciprocating (or) translating follower:-

When follower reciprocates in guides as cam rotates uniformly, it is known as R/T follower. fig's (a) to (d) are called R/T followers.

(b). Oscillating (or) rotating follower:-

When uniform rotary motion of cam is converted into predetermined oscillatory motion of

follower, it is called oscillating follower. (fig. (e))

*3). According to path of motion of follower:-

(a). Radial follower:-

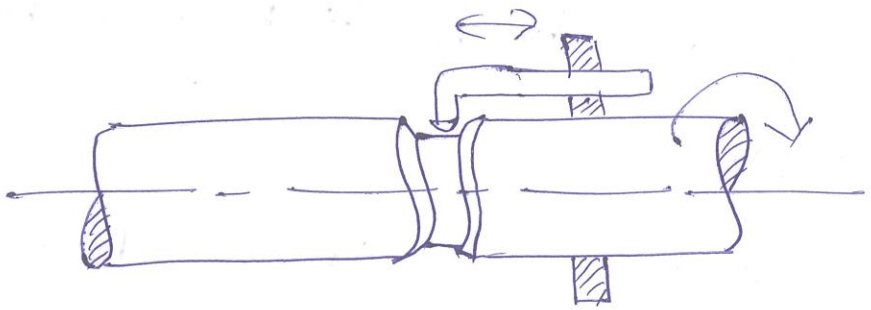
When motion of follower is along anaxis passing through centre of cam, it is called R. follower. (fig's. (a) to (e)).

(b). offset follower:-

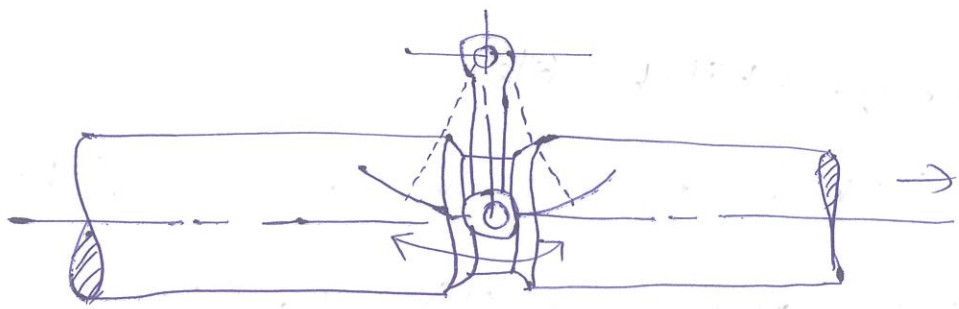
When motion of follower is along an axis away form axis of cam centre, it is called off-set follower. (fig. (f)).

* classification of cams:-

1). Radial (or) disc cam:-



→ cylindrical cam with R.F



→ cylindrical cam with O.F

fig. (A) cylindrical cam.

In radial cams, follower reciprocates (or) oscillates in a direction ⊥ to cam axis. (fig's 20.1(a) to (f)).

## 2). Cylindrical cam :-

In this, follower reciprocates (or) oscillates in a direction || to cam axis. (fig (A)).

## * Terms used in Radial cams :-

→ 1). Base circle :- It is the smallest circle that can be drawn to the cam profile.

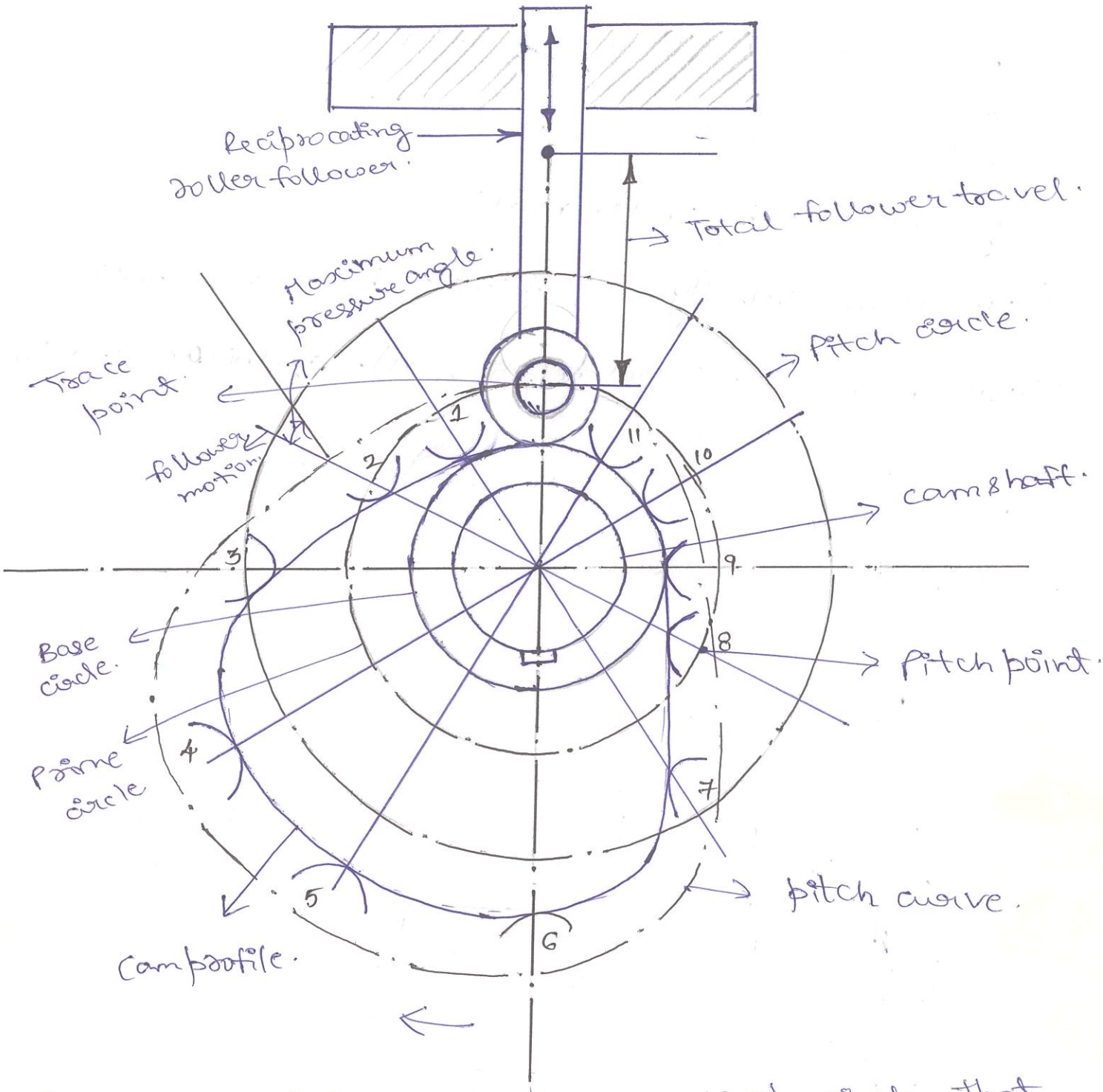
→ 2). Trace pt. :- It is a reference pt. on follower & is used to generate pitch curve.

→ 3). Pressure Angle :- Angle b/w direction of follower motion & normal to pitch curve.

→ 4). Pitch pt. :- It is a pt. on pitch curve having max. pressure angle.

→ 5). Pitch circle :- It is a circle drawn from centre of cam through pitch pts.

→ 6). Pitch curve :- It is curve generated by trace pts as follower moves relative to cam.




→7). Prime circle :- It is the smallest circle that can be drawn from centre of cam & tangent to pitch curve.

→8). Lift (or) stroke :- It is max. travel of the follower from its lowest position to the topmost position.

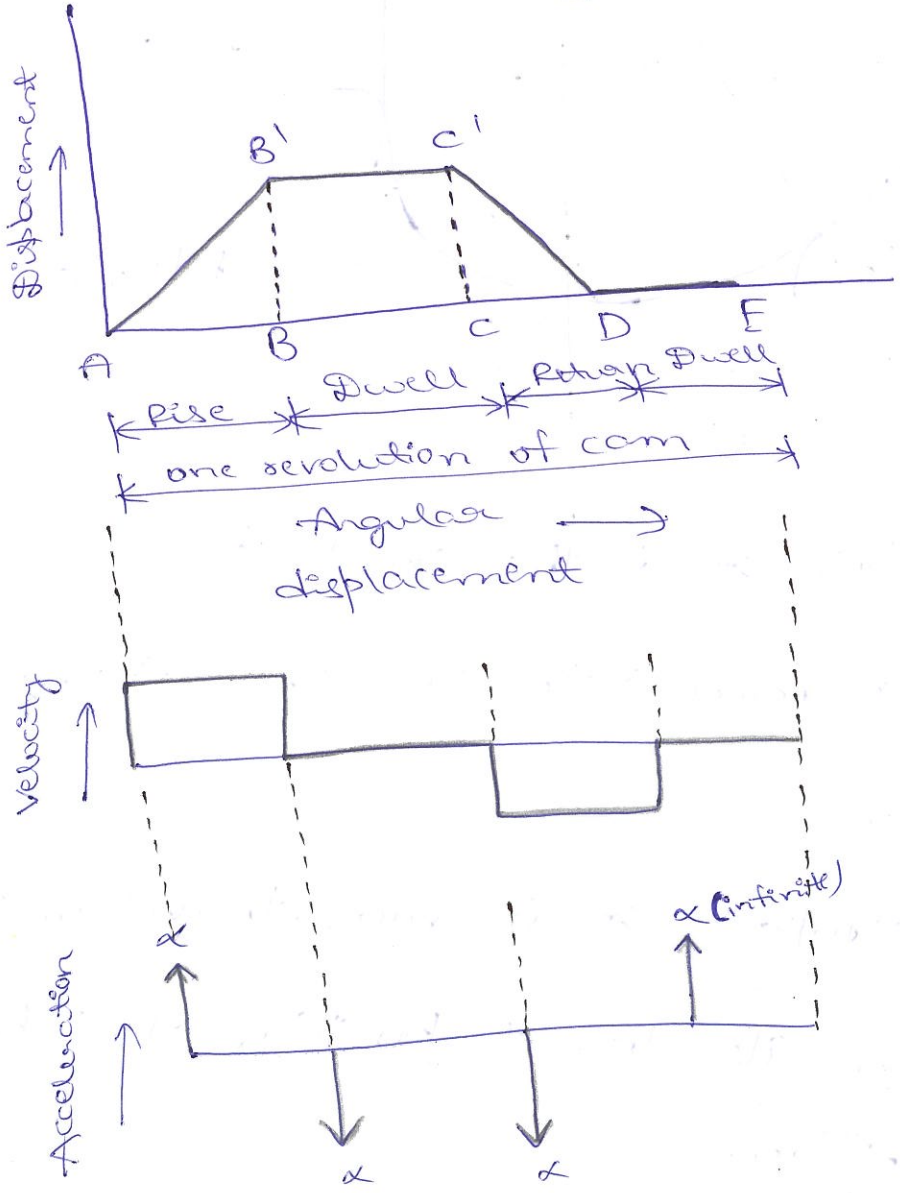
## * Motion of the follower's

During its travel, it (follower) will have any one of the foll. motions —

1). uniform velocity 2). Simple harmonic motion.

3). uniform acc. & retardation 4). cycloidal motion.

## * Displacement, Velocity & Acceleration Dgms when follower moves with uniform velocity :




Displacement Dgm

Velocity Dgm

Acceleration Dgm

→ Abscissa (base) represents time (or) 'θ' of cam in degrees.

→ ordinate represents displacement, velocity & acceleration of follower.

→ Since follower moves with uniform velocity during its rise & return stroke, therefore displacement dgm consists of straight lines.

→ Follower remains at rest during part of cam rotation, which are known as dwell periods.

* Displacement, velocity & Acceleration Dgms when the follower moves with SHM:-

→ Displacement Dgm steps:-

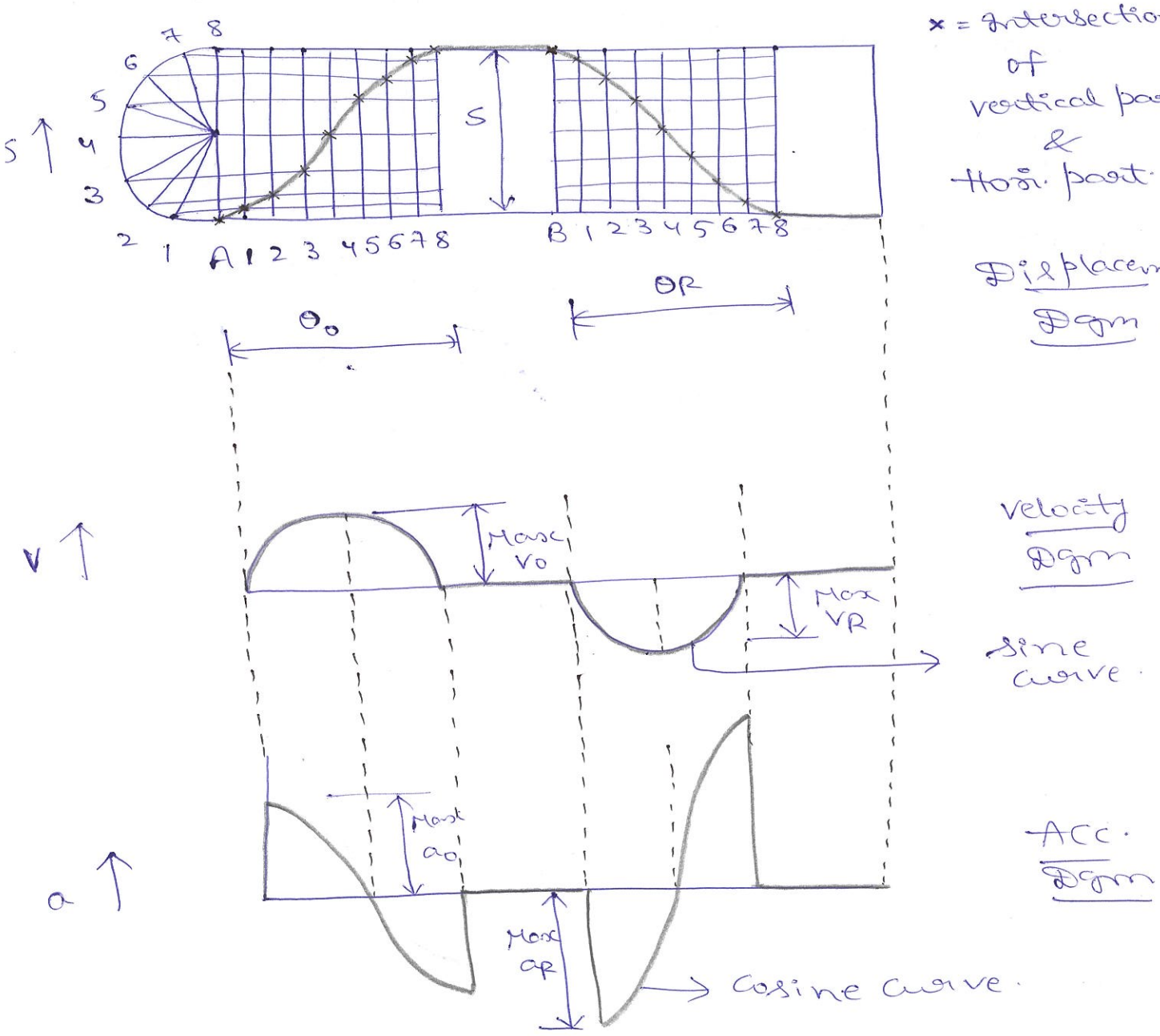
1). Draw a semi-circle on follower stroke as Dia

2). Divide semi-circle into any no. of even equal parts (say eight).

3). Divide 'θ' of cam during out stroke & return stroke into same no. of equal parts.

4). Displacement Dgm is obtained by projected points.

→ Since follower moves with SHM, therefore velocity Dgm consists of sine curve & Acc. Dgm is a cosine curve.




→ Time required for outstroke of follower in sec, $t_o = \frac{\theta_o}{\omega}$ ✓

→ Time required for return stroke of follower in sec, $t_R = \frac{\theta_R}{\omega}$ ✓

→ During outstroke, Max. velocity of follower,

$$V_o \ (V_{pt}) = \frac{\pi \omega S}{2\theta_o}$$ ✓

→ During outstroke, Max acc. of follower,

$$a_o \ (a_{pt}) = \frac{\pi^2 \omega^2 S}{2(\theta_o)^2}$$ ✓

→ During ~~return stroke~~, Max. velocity of follower, $V_R = \frac{\pi \omega S}{2\theta_R}$ ✓

→ During return stroke, Max. acc. of follower

$$a_R = \frac{\pi^2 \omega^2 S}{2(\theta_R)^2}$$ ✓

* Displacement, Velocity & Acceleration Dgms when follower moves with uniform Acceleration & Retardation:-

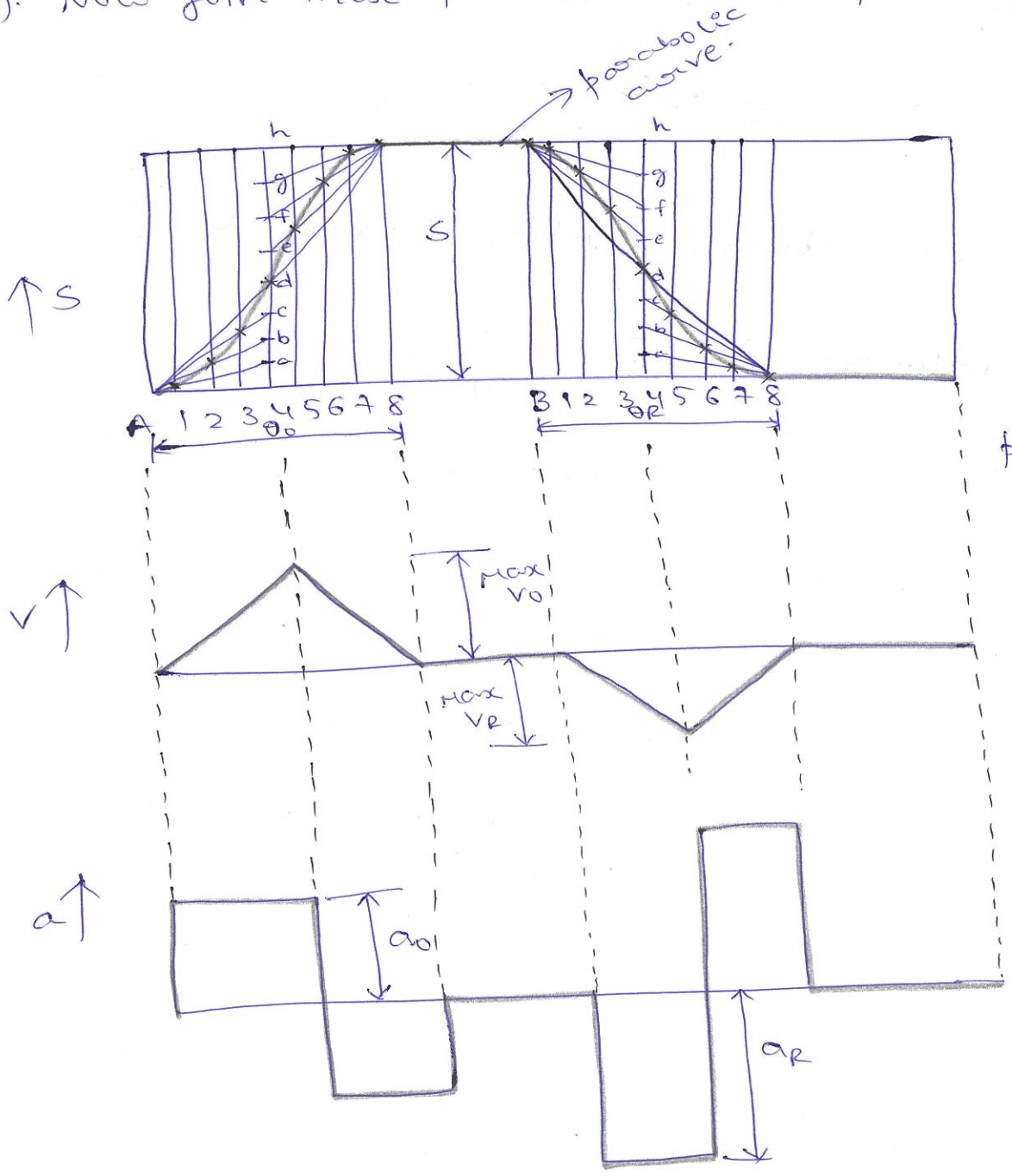
→ Displacement Dgm steps:-

1). Divide 'θ' of cam during outstroke ($\theta_o$) into any even no of equal parts (say 8) & draw vertical lines through these points.

2). Divide stroke of follower (S) into same no. of equal even parts.

3). Join Aa to intersect vertical line through pt.1 at B. Similarly, obtain other pts. C, D, ...

4). Now join these pts. to obtain parabolic curve.




$$\rightarrow \quad t_o = \frac{\theta_o}{\omega} \quad , \quad t_R = \frac{\theta_R}{\omega}$$

$$\rightarrow \quad V_{o(max)} = \frac{2\omega S}{\theta_o} \quad , \quad V_{R(max)} = \frac{2\omega S}{\theta_R}$$

$$\rightarrow \quad a_{o(max)} = \frac{4\omega^2 S}{(\theta_o)^2} \quad , \quad a_{R(max)} = \frac{4\omega^2 S}{(\theta_R)^2}$$

* construction of cam profile for a radial cam:

→ In order to draw cam profile, first of all displacement dgm is drawn.

→ In constructing cam profile, cam is imagined to be stationary & follower is allowed to rotate in opp. direction to cam rotation.

* A cam is to give following motion to a knife-edged follower:

1). outstroke during 60° of cam rotation

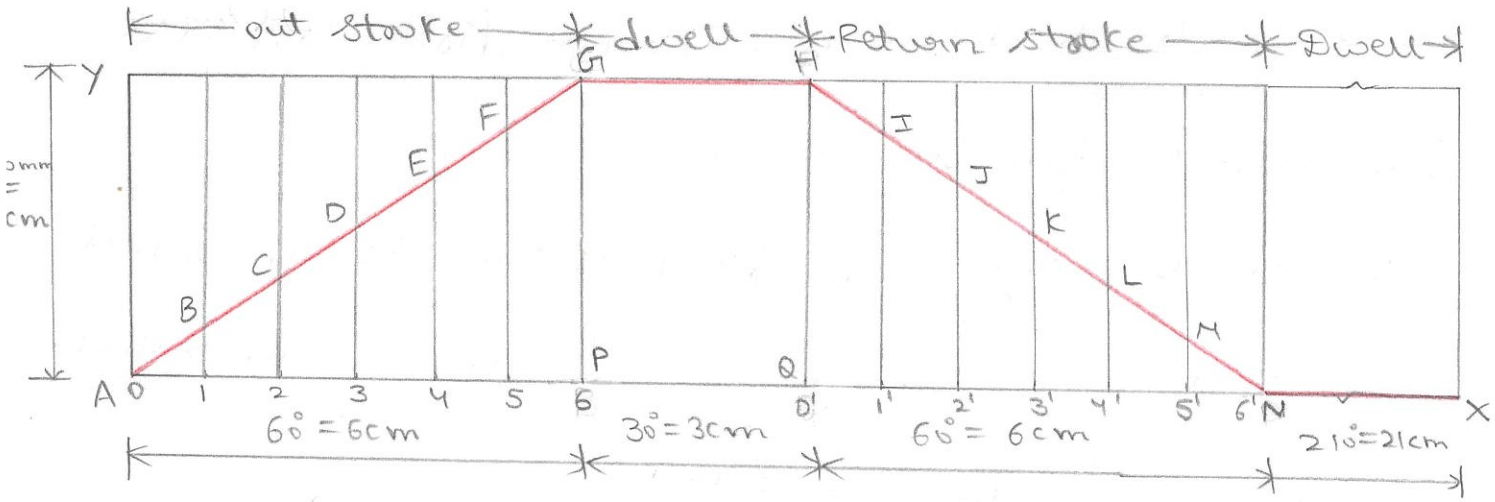
2). Dwell for next 30° of cam rotation

3). Return stroke during next 60° of cam rotation

4). Dwell for remaining 210° of cam rotation.

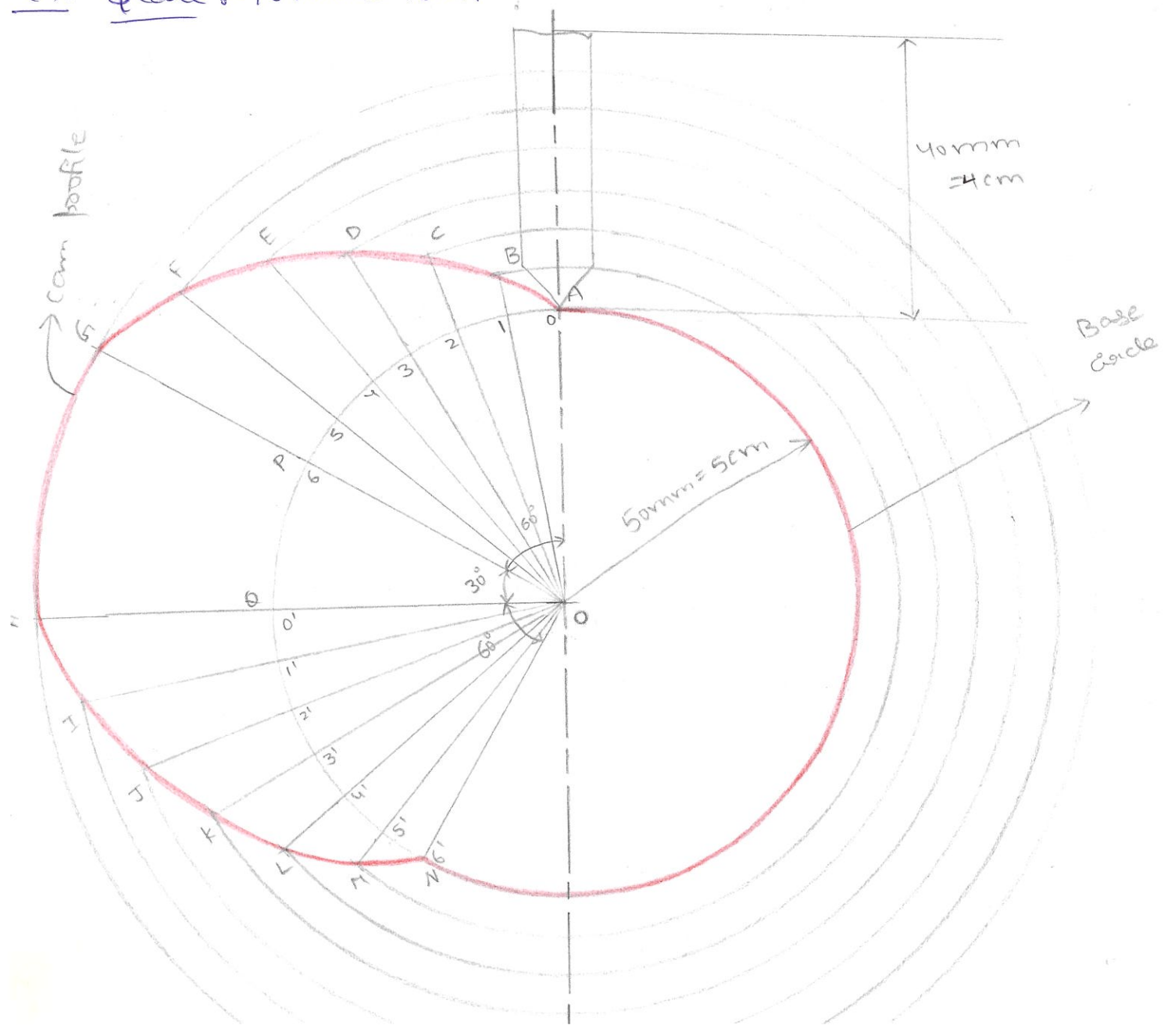
The stroke of follower is 40mm & min. radius of cam is 50mm. The follower moves with uniform velocity during both outstroke & return strokes. Draw profile of cam when (a). axis of follower passes through axis of cam shaft. & (b). axis of follower is offset by 20mm from axis of cam shaft.

→ Displacement Dgm :- Scale: 10° = 1cm & 10mm = 1cm.



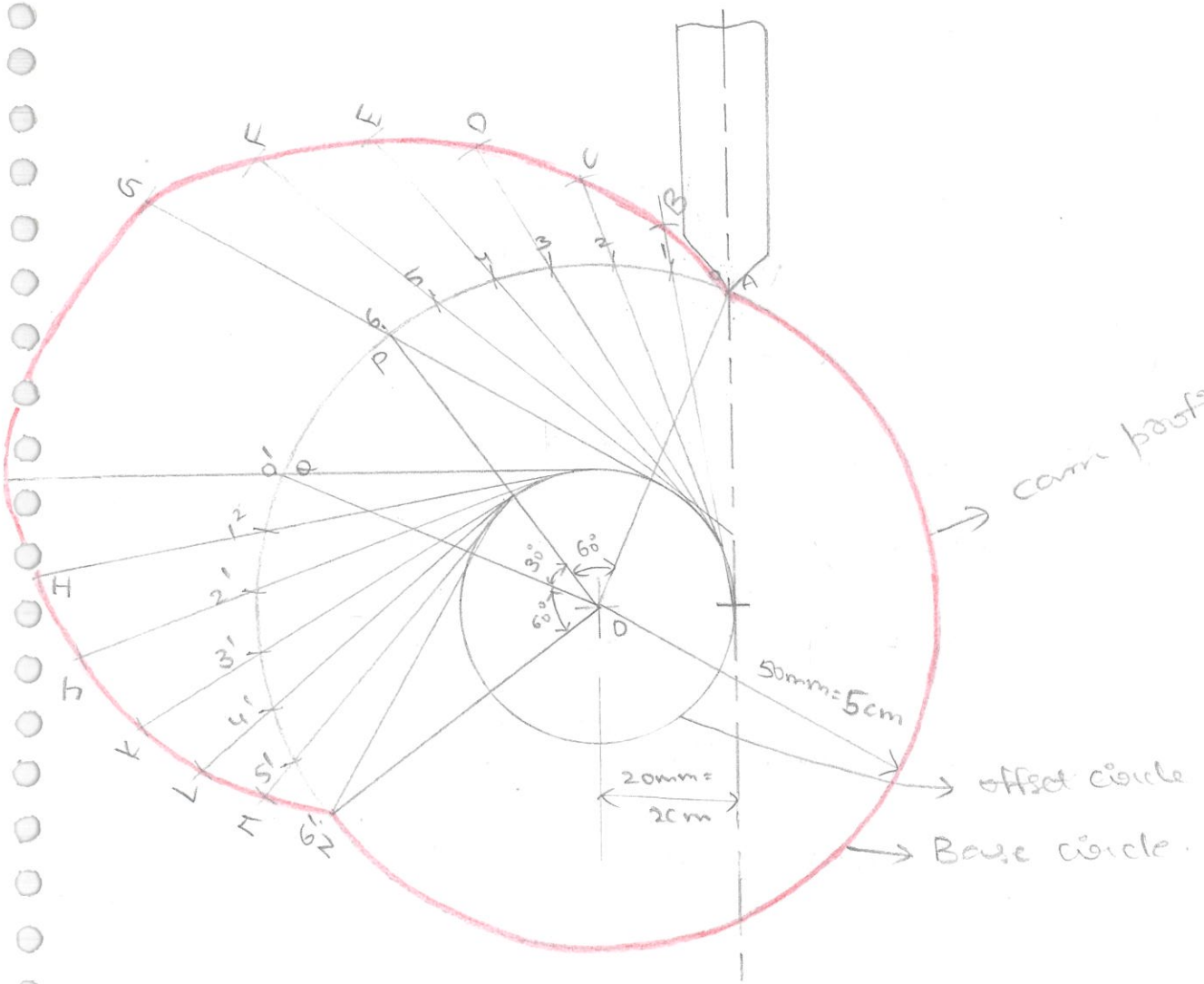

∴ complete displacement dgm is shown by AGHNX.

(a). Scale :- 10mm = 1cm



∴ The curve AGHNA is complete profile of the cam.

(b) Scale :- 10mm = 1cm.




∴ The curve AGHNA is complete profile of the cam

* A cam is to be designed for a knife edge follower with following data :

1). cam lift = 40mm during 90° of cam rotation with SHM. 2). Dwell for next 30°.

3). During next 60° of cam rotation, follower returns to its original position with SHM

4). Dwell during remaining 180°.

Draw profile of cam when —

(a). line of stroke of follower passes through axis of cam shaft & (b). line of stroke is offset 20mm from the axis of cam shaft.

The radius of base circle of cam is 40mm. Determine max. Velocity & acc. of follower during its ascent & descent, if cam rotates at 240rpm.

→ $S = 40mm = 0.04m$, $\theta_0 = 90° \times \frac{\pi}{180} = 1.571 rad$,

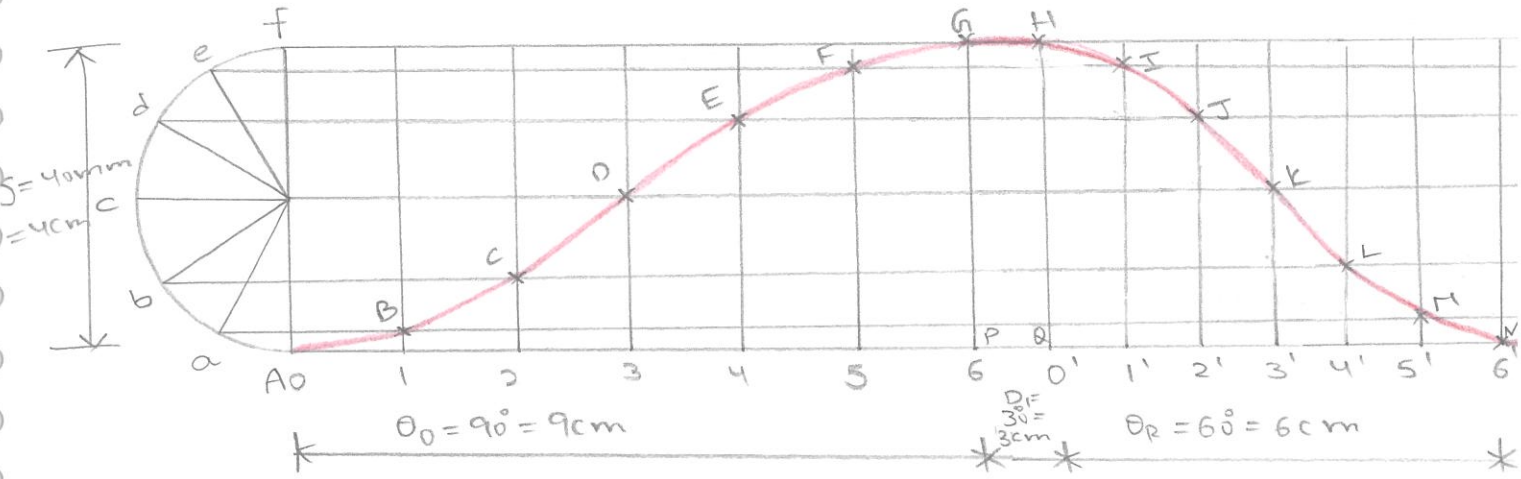
$\theta_R = 60° \times \frac{\pi}{180} = 1.047 rad$, $N = 240 rpm$.

Scale :- 10mm = 1cm

10° = 1cm

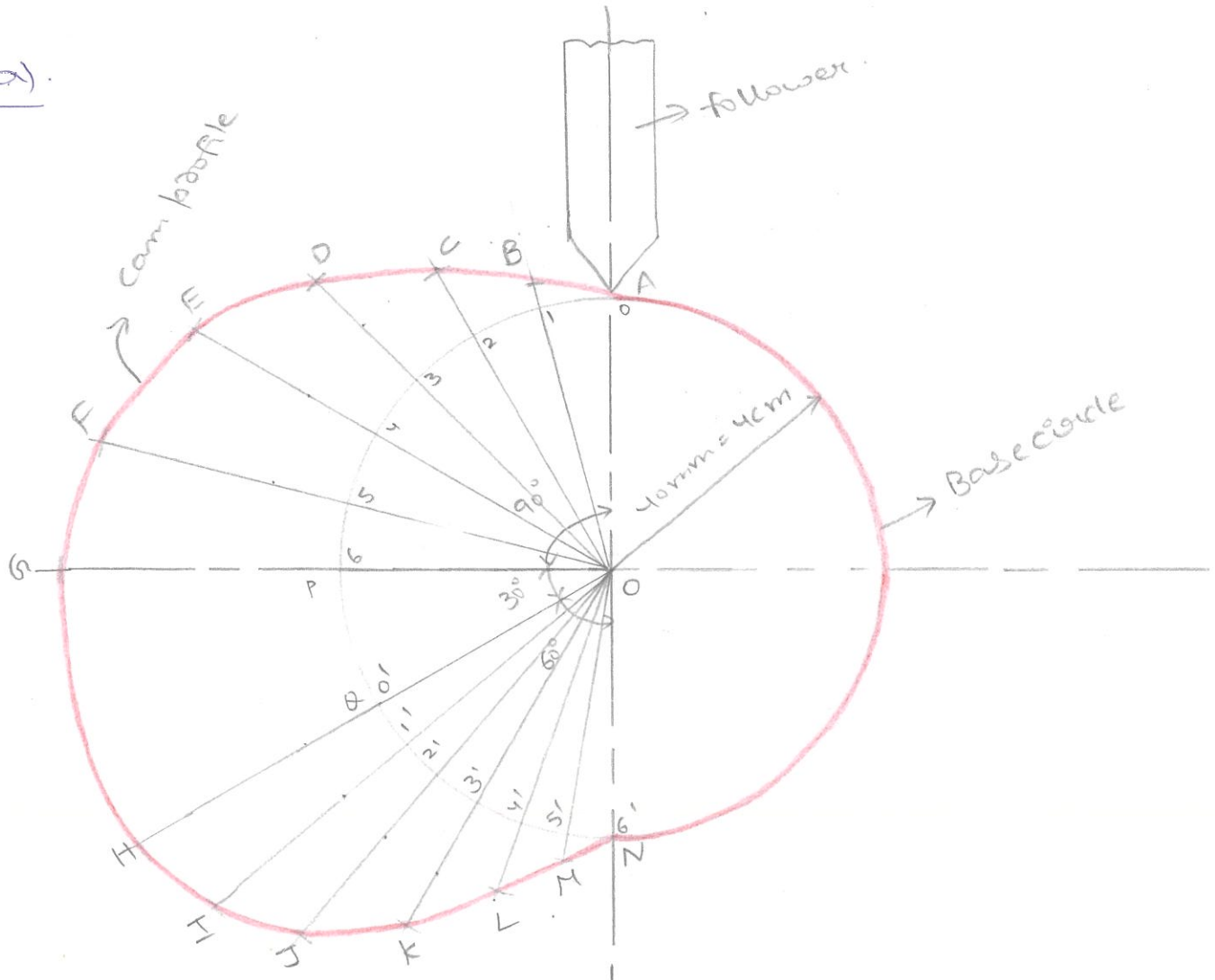
Displacement Dgm :-

$\frac{S}{\phi}$ :- 10° = 1cm, 10mm = 1cm



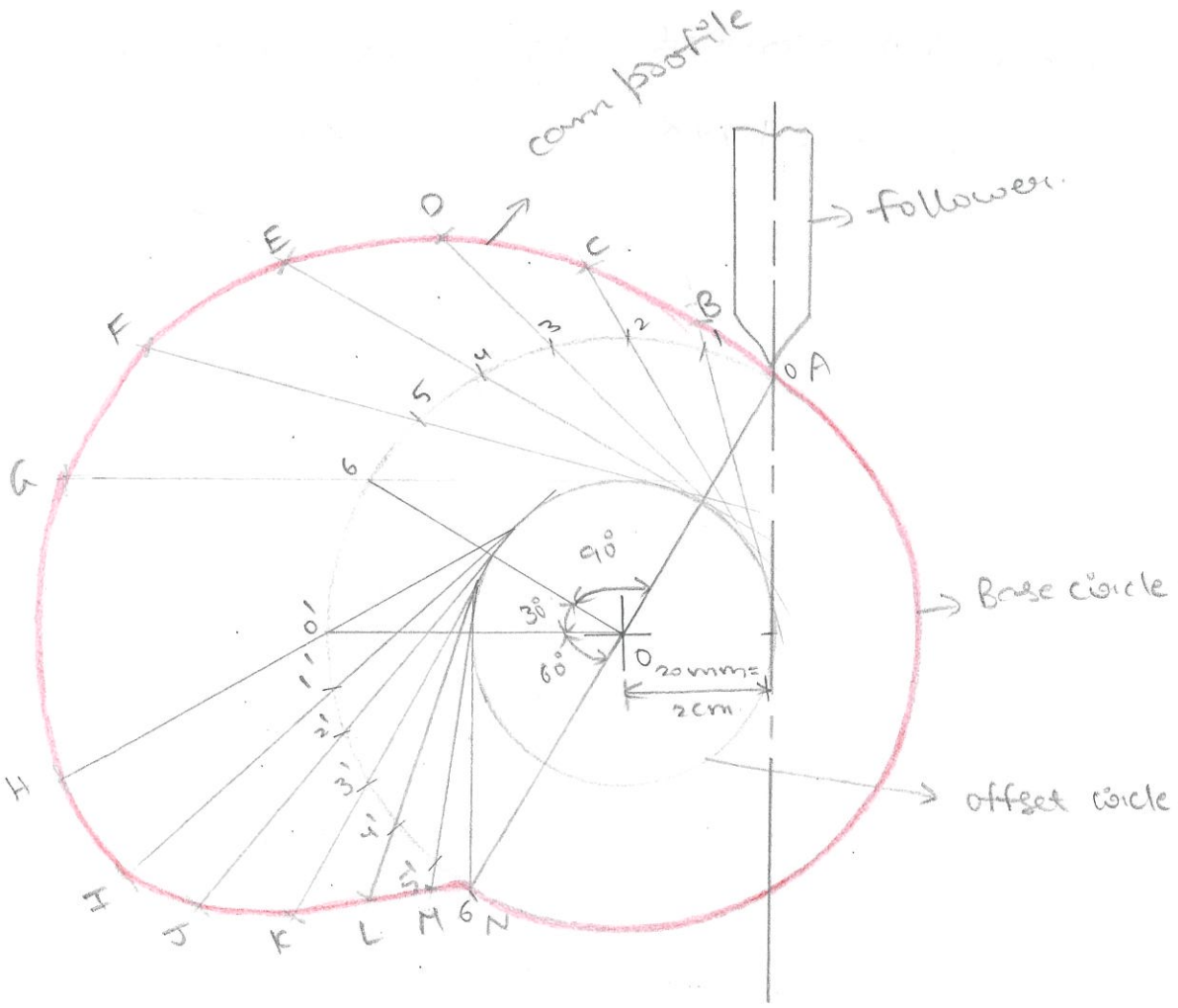

∴ complete displacement dgm is shown by AGHNX.

(a).



∴ The curve AGHNA is complete profile of the cam.

(b). Scale :- 10 mm = 1 cm.




∴ The curve AGHNA is complete profile of the cam.

Max. velocity of follower during its ascent & descent ($V_o$ & $V_R$) :-

$$\Rightarrow \omega = \frac{2\pi N}{60} = \frac{2\pi 240}{60} = 25.14 \text{ m/sec.}$$

$$\Rightarrow V_o = \frac{\pi \omega s}{2\theta_o} = \frac{\pi \times 25.14 \times 0.04}{2 \times 1.571} = 1 \text{ m/sec} \checkmark$$

$$\Rightarrow V_R = \frac{\pi \omega s}{2\theta_R} = \frac{\pi \times 25.14 \times 0.04}{2 \times 1.047} = 1.51 \text{ m/sec} \checkmark$$

Max. acc. of follower during its ascent & descent ($a_o$ & $a_R$) :-

$$\Rightarrow a_o = \frac{\pi^2 \omega^2 s}{2(\theta_o)^2} = 50.6 \text{ m/sec}^2 \checkmark$$

$$\Rightarrow a_R = \frac{\pi^2 \omega^2 s}{2(\theta_R)^2} = 113.8 \text{ m/sec}^2 \checkmark$$

~~*~~ A cam rotating clockwise at a uniform speed of 1000 rpm. is required to give a roller follower the motion defined below :-

1). Follower to move outwards through 50mm during 120° of cam rotation,

2). Follower to dwell for next 60° of cam rotation.

3). Follower to return to its starting position during next 90° of cam rotation.

4). Follower to dwell for rest of cam rotation.

The minimum radius of cam is 50 mm & dia. of roller is 10mm. The line of stroke of follower is offset by 20 mm from axis of cam shaft. If displacement of follower takes place with uniform & equal acc. & retardation on both outward & return strokes, draw profile of the cam & find max. velocity & acc. during both strokes.

→ $N = 1000$ rpm, $S = 50\text{ mm} = 0.05\text{ m}$, $\theta_0 = 120^\circ \times \frac{\pi}{180}$

$\Rightarrow \theta_0 = \frac{2\pi}{3}\text{ rad} = 2.1\text{ rad}$, $\theta_R = 90^\circ \times \frac{\pi}{180} = 1.571\text{ rad}$.

$r_{cam} = 50\text{ mm}$, $r_{prime\ circle} = r_{cam} + r_{roller} = 50 + 5 = 55\text{mm}$

$\Rightarrow \omega = \frac{2\pi N}{60} = \frac{2\pi 1000}{60} = 104.7\text{ rad/sec}$ ✓

$\Rightarrow$ Max. $V_0 = \frac{2\omega S}{\theta_0} = 5\text{ m/sec}$ ✓

$\Rightarrow$ Max. $V_R = \frac{2\omega S}{\theta_R} = 6.66\text{ m/sec}$ ✓

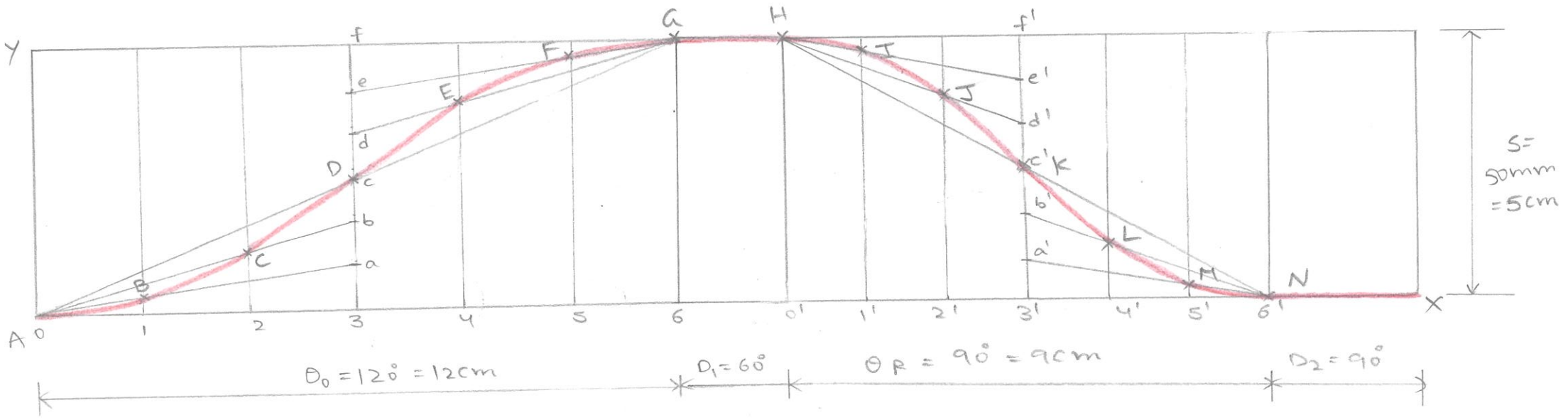
$\Rightarrow$ Max. $a_0 = \frac{4\omega^2 S}{(\theta_0)^2} = 497.2\text{ m/sec}^2$ ✓

$\Rightarrow$ Max. $a_R = \frac{4\omega^2 S}{(\theta_R)^2} = 888\text{ m/sec}^2$

1

Displacement Dgm :- Scale :- 10mm = 1cm

10° = 1cm

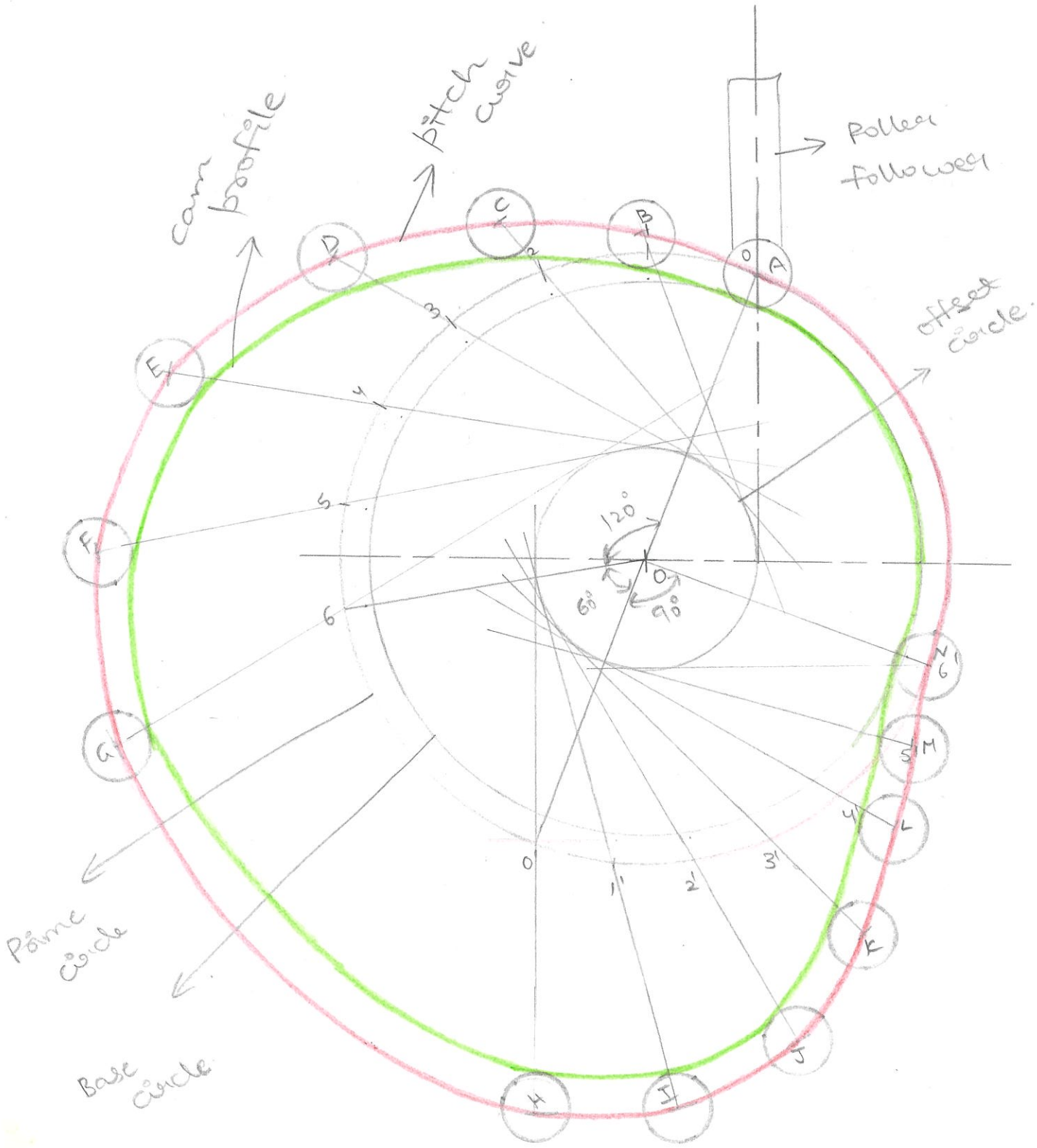



∴ complete displacement dgm is shown by

A G H N X.

Cam profile dgm:- Scale :- 10mm = 1cm.

∴ curve AGHNA is pitch curve & below curve to pitch curve is cam profile.

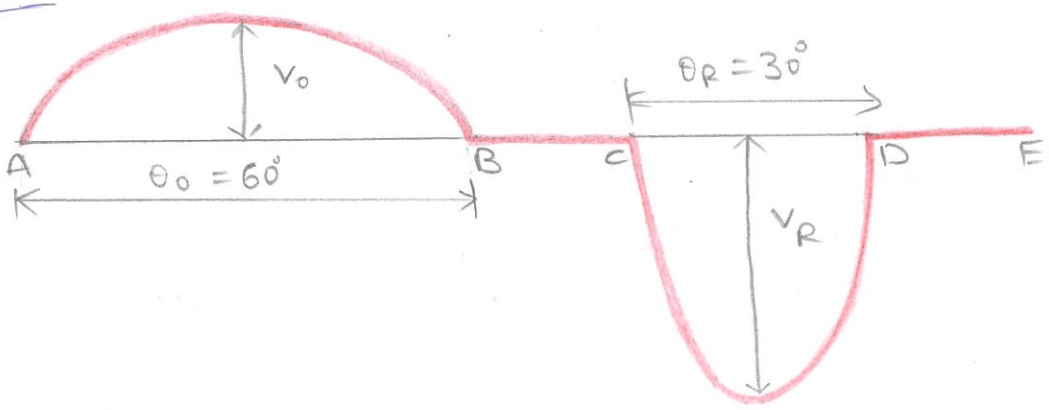


* Draw velocity dgm & Acc. dgm when follower moves with SHM for following data :-

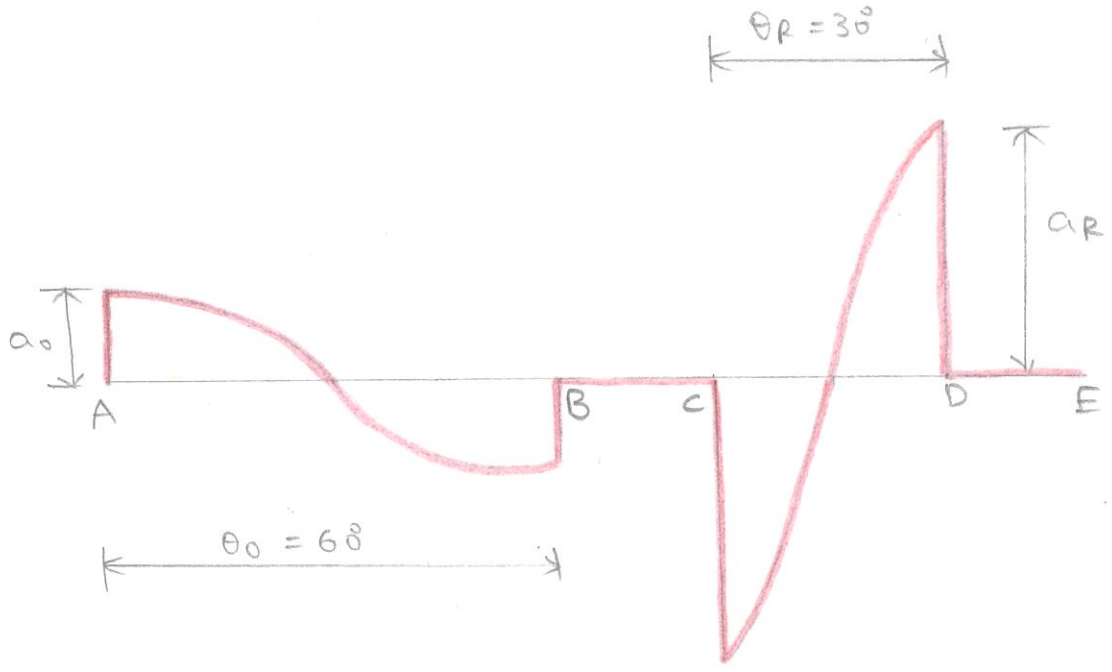
$V_o = 0.39$ m/sec, $V_R = 0.785$ m/sec, $\theta_o = 60°$

$a_o = 6.13$ m/sec$^2$, $a_R = 24.67$ m/sec$^2$, $\theta_R = 30°$

→ Scale :- $10° = 1$ cm.




Velocity Dgm - ABCDE



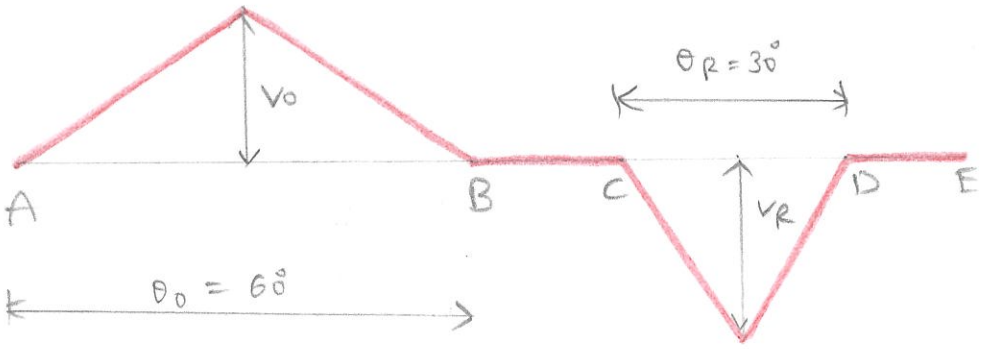

Acceleration Dgm - ABCDE

* Draw Velocity dgm & ACC. dgm when follower moves with uniform acc. & ret. for following data:

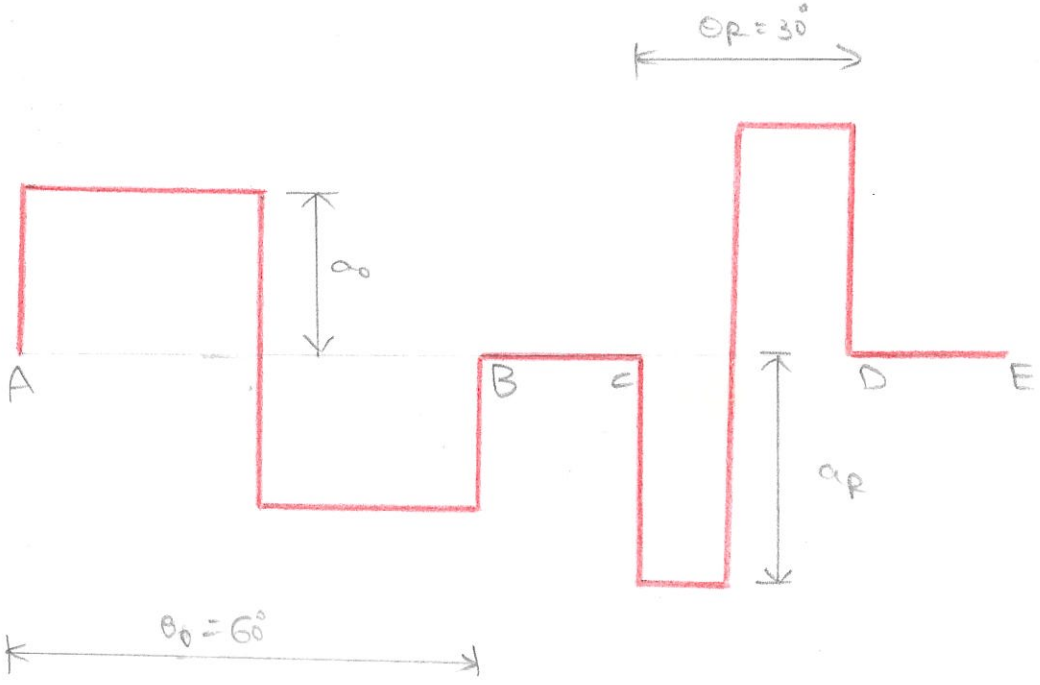
$V_o = 4.32 m/sec$, $V_R = 4.8 m/sec$, $a_o = 4.67 m/sec^2$,

$a_R = 5.76 m/sec^2$, $\theta_o = 60°$, $\theta_R = 30°$.

→ Scale: $10° = 1cm$.



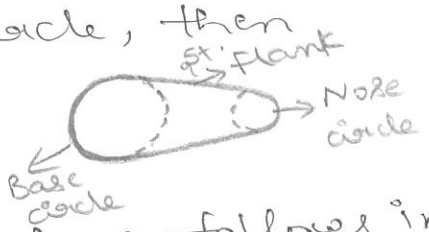

Velocity Dgm – ABCDE




Acceleration Dgm – ABCDE.

## * Tangent cam with reciprocating roller follower:-

→ When the flanks of cam are straight & tangential to base circle & nose circle, then cam is known as "tangent cam".

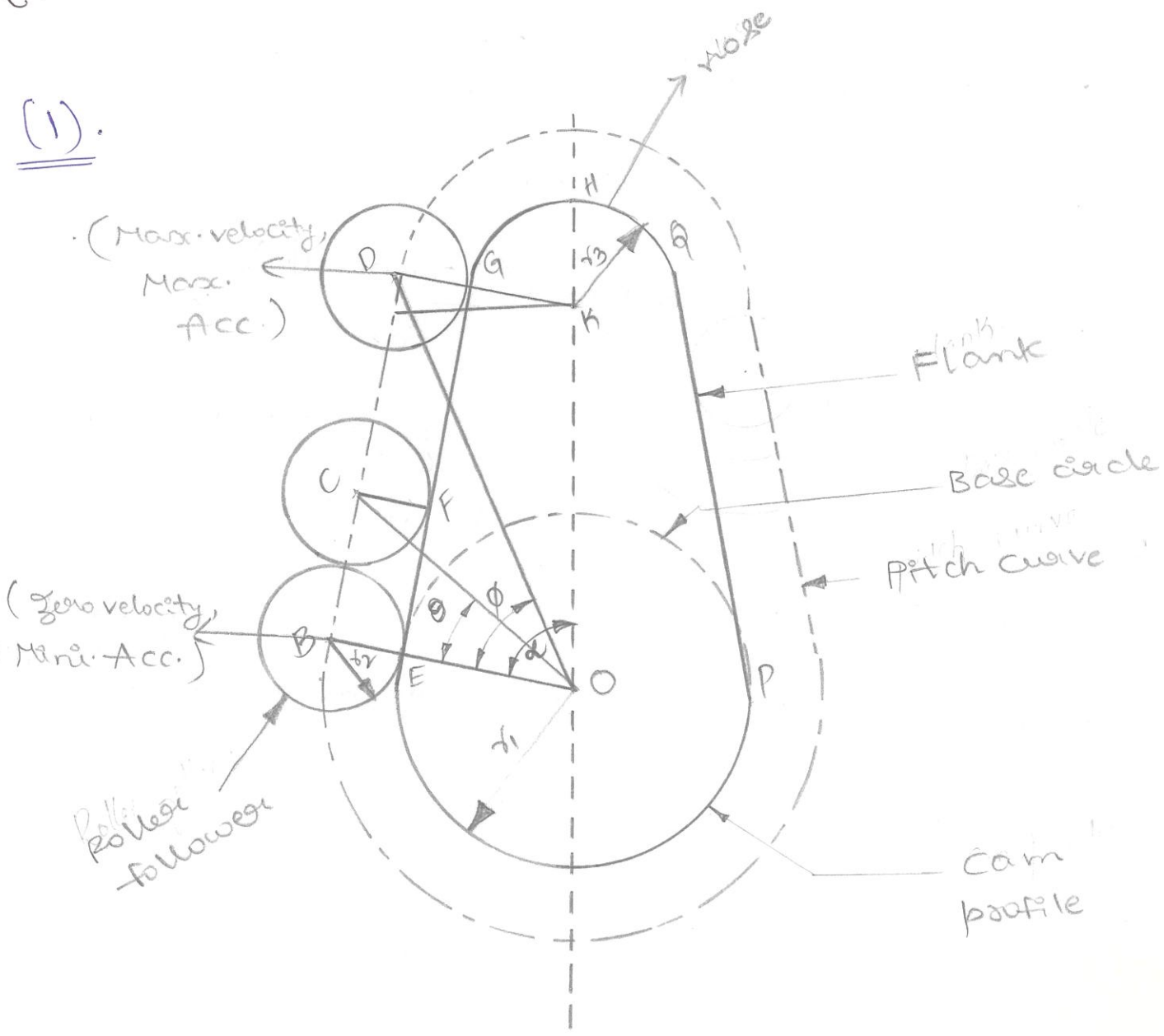



→ S, V, A of follower is obtained as follows in these two cases :-

(1). When roller has contact with st. flanks

(2). When roller has contact with nose.

(1).



Here, $r_1$ = radius of base circle, $r_2$ = Radius of roller,

$r_3$ = radius of nose circle, $\alpha$ = angle of ascent,

$\theta$ = roller displacement, $\phi$ = Angle b/w contact of roller and flank.

$$\Rightarrow S = (r_1 + r_2)\left(\frac{1-\cos\theta}{\cos\theta}\right) \checkmark$$

$$\Rightarrow V = \omega(r_1 + r_2)\left(\frac{\sin\theta}{\cos^2\theta}\right) \checkmark$$

$$\Rightarrow V_{max} = \omega(r_1 + r_2)\left(\frac{\sin\phi}{\cos^2\phi}\right) \checkmark$$

$$\Rightarrow a = \omega^2(r_1 + r_2)\left(\frac{2-\cos^2\theta}{\cos^3\theta}\right) \checkmark$$

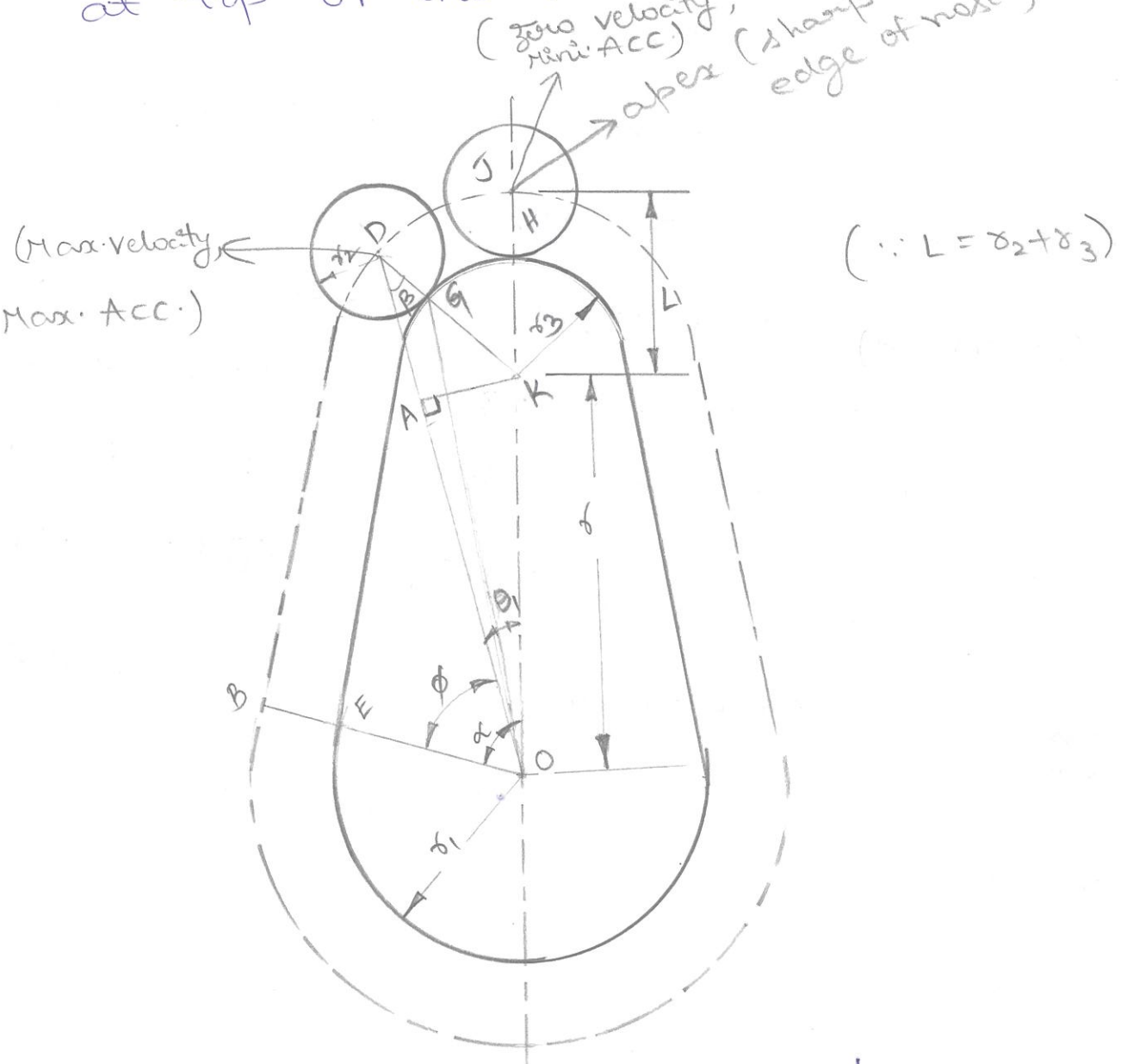
$$\Rightarrow a_{min} = \omega^2(r_1 + r_2) \checkmark$$

$$\Rightarrow a_{max} = \omega^2(r_1 + r_2)\left(\frac{2-\cos^2\phi}{\cos^3\phi}\right) \checkmark$$

→ When roller is at beginning of its lift (or) roller has contact with flank at E, we will get min. acc.

→ When roller just leaves contact with flank at A (or) when flank merges into nose, we will get max acc.

(2). Here, $\theta_1$ = Angle turned by cam measured from position when roller is at top of the nose.




$$\Rightarrow S = L + r - r\cos\theta_1 - \left(L^2 - r^2\sin^2\theta_1\right)^{\frac{1}{2}}$$

$$\Rightarrow V = \omega r\left[\sin\theta_1 + \frac{r\sin 2\theta_1}{2\left(L^2 - r^2\sin^2\theta_1\right)^{1/2}}\right].$$

$$\Rightarrow a = \omega^2 r\left[\cos\theta_1 + \frac{L^2 r\cos 2\theta_1 + r^3\sin^4\theta_1}{\left(L^2 - r^2\sin^2\theta_1\right)^{3/2}}\right]$$

→ Velocity is '0' at H & 'max' at A

→ Acc. is 'mini' at H & 'max' at A.

* In a symmetrical tanget cam operating a roller follower, the least radius of cam is 30 mm & roller radius is 17.5 mm. The angle of ascent is 75° & total lift is 17.5 mm. The speed of cam shaft is 600 rpm. calculate the Accelerations of the follower at beginning of lift, where straight flank merges into circular nose & at apex circular nose. Assume that there is no dwell b/w ascent & descent. ($\phi = 25.6°$) (radius of nose is 23.88 mm)

→ $r_1 = 30$ mm, $r_2 = 17.5$, $\alpha = 75°$, T.Lift = 17.5 mm,

$r_3 = 23.88$ mm, $N = 600$ rpm ⇒ $\omega = \frac{2\pi N}{60} = 62.84$ rad/sec.

→ Acc. at beginning of lift will come in (1) case,

⇒ $a_{max} = \omega^2 (r_1 + r_2)$

$= 187600$ mm/sec$^2$

$\therefore a_{max} = 187.6$ m/sec$^2$ ✓

→ Acc. where st. flank merges into circular nose will come in (1) case,

⇒ $a_{max} = \omega^2 (r_1 + r_2) \left[ \frac{2 - \cos^2\phi}{\cos^3\phi} \right]$

$= 303800 \text{ mm/sec}^2$

$\therefore a_{max.} = 303.8 \text{ m/sec}^2$ ✓

→ Acc. at apex of circular nose will come

in (2) case, $a = \omega^2 r \left[\cos\theta_1 + \frac{L^2 r \cos 2\theta_1 + r^3 \sin^4\theta_1}{(L^2 - r^2\sin^2\theta_1)^{3/2}}\right]$

Here $\theta_1 = 0$ as roller axis lies on cam axis

$$\Rightarrow a = \omega^2 r \left[1 + \frac{L^2 r (1) + r^3 (0)}{(L^2 - 0)^{3/2}}\right]$$

$$= \omega^2 r \left(1 + \frac{L^2 r}{(L^2)^{3/2}}\right) = \omega^2 r \left(1 + \frac{r}{L}\right).$$

$$= (62.84)^2 \times (23.62)\left(1 + \frac{23.62}{17.5 + 23.88}\right)$$

$(\because L = r_2 + r_3 = 17.5 + 23.88 \text{ mm})$

$= 146\,530 \text{ mm/sec}^2$

$\therefore a = 146.53 \text{ m/sec}^2$ ✓

* Circular Arc cam with flat-faced follower :-

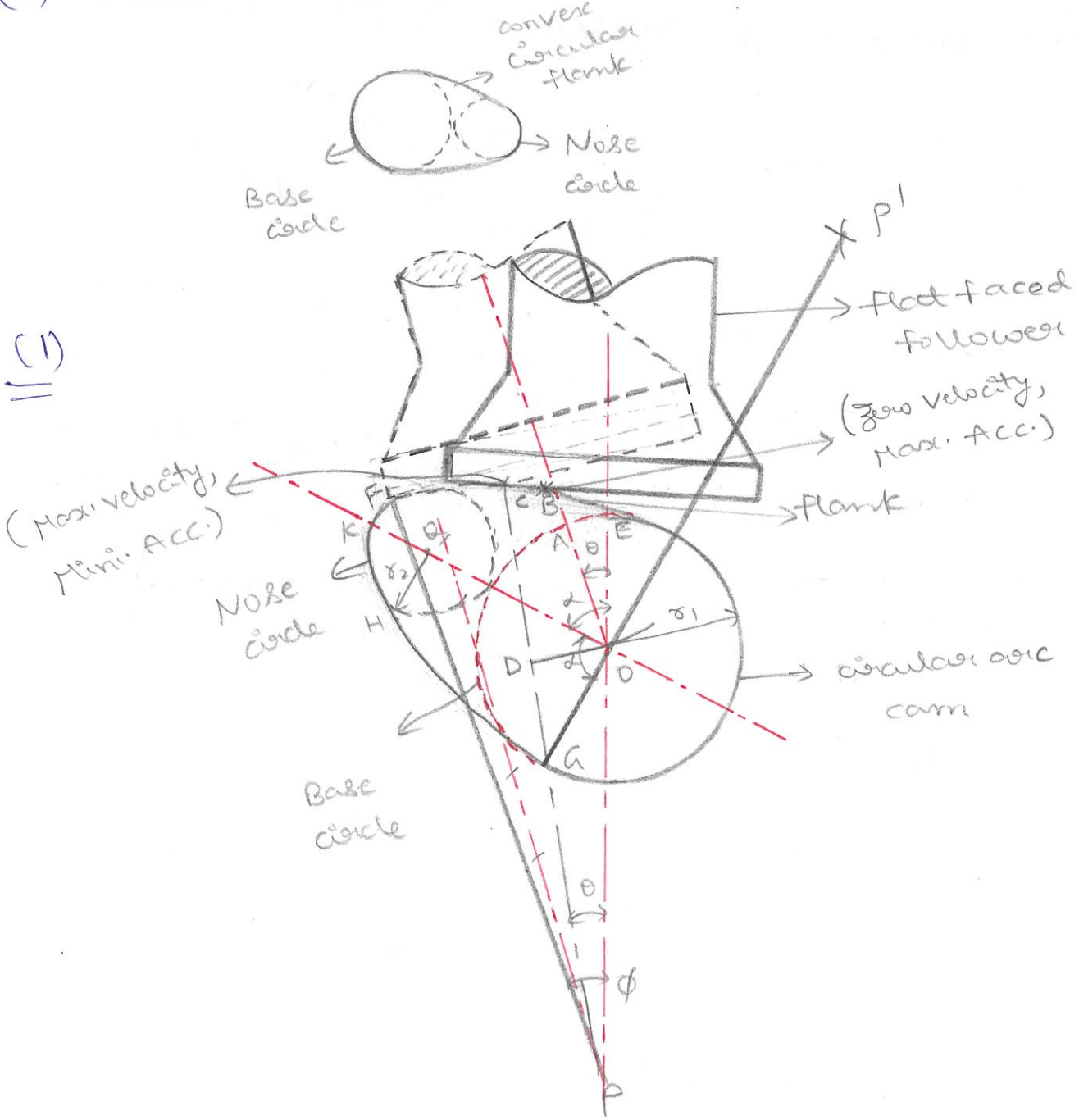
→ When flanks of cam connecting base circle & nose are of convex circular arcs, then cam is known as "circular arc cam".

→ S, V, A of follower is obtained as follows in these two cases :-

(1). When flat face of follower has contact on circular flank

(2). When flat face of follower has contact on nose.

(1)




(OQ = Distance b/w cam centre & nose centre)

Here, $r_1$ = radius of base circle, $r_2$ = radius of nose circle, $R$ = radius of circular flank, $\alpha$ = angle of ascent, $\phi$ = angle of action of cam on circular flank, $2\alpha$ = total angle of action of cam.

$\Rightarrow \quad s = (R - r_1)(1 - \cos\theta)$

$\Rightarrow \quad V = \omega (R - r_1) \sin\theta$

$\Rightarrow \quad V_{max} = \omega (R - r_1) \cos\theta$

$\Rightarrow \quad a = \omega^2 (R - r_1) \cos\theta$

$\Rightarrow \quad a_{min} = \omega^2 (R - r_1) \cos\phi$

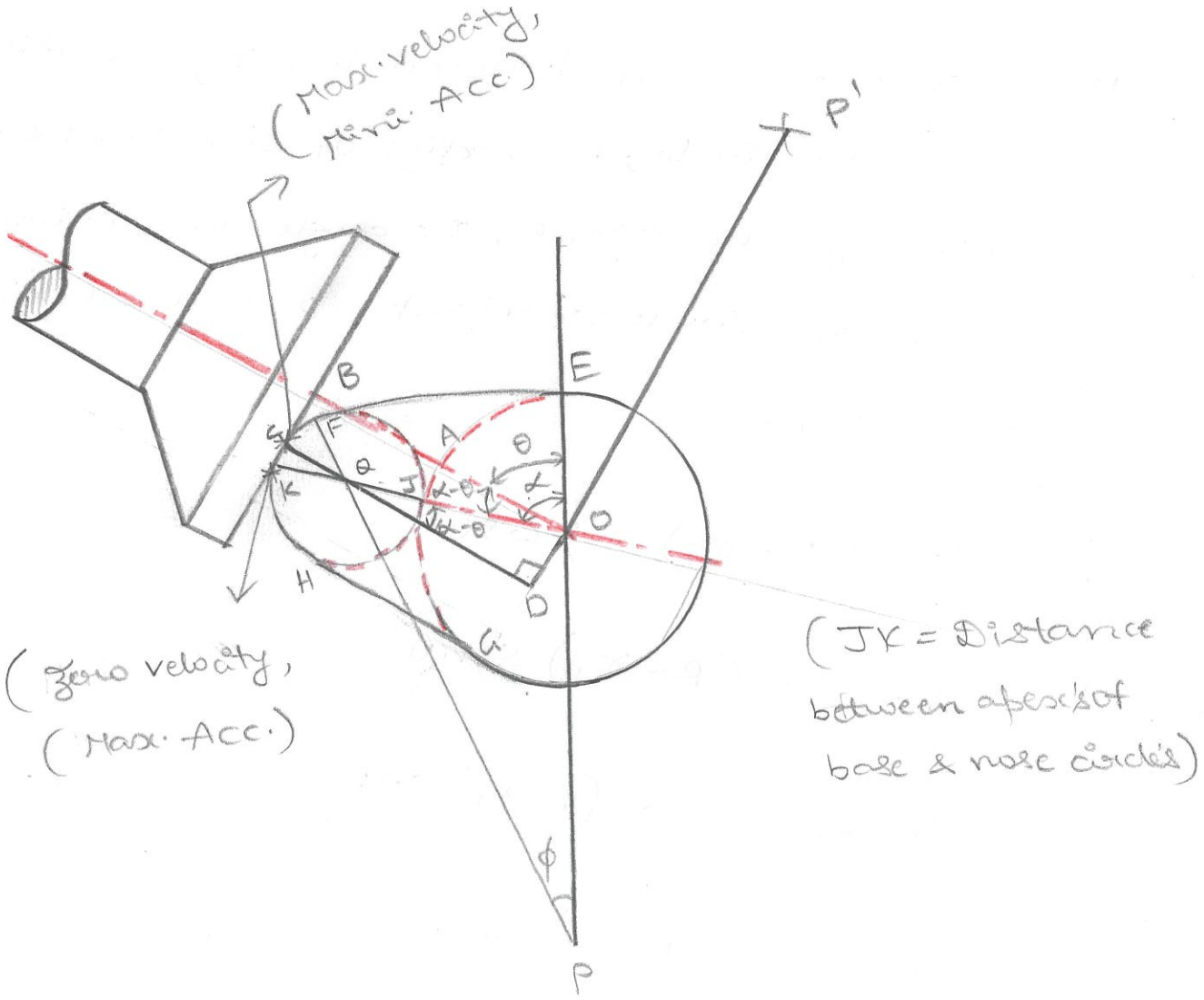
$\Rightarrow \quad a_{max} = \omega^2 (R - r_1)$

→ When follower contact just shifts from circular flank to circular nose, velocity will be maximum & '0' at beginning of ascent.

→ At beginning of ascent, acceleration is maximum & min. when $\theta = \phi$.

(2). $\Rightarrow s = r_2 + OQ - r_1$

$\Rightarrow V = \omega OQ \sin(\alpha - \theta)$




$\Rightarrow a = -\omega^2 \times OQ \times \cos(\alpha - \theta)$

$\Rightarrow$ Max. retardation $= -\omega^2 OQ$

$\Rightarrow$ Mini. retardation $= -\omega^2 OQ \cos(\alpha - \phi)$.

→ Velocity is zero when follower is at apex of nose & max. when follower changes contact from circular flank to circular nose.

→ Acc. is mini. when follower changes contact from circular flank to circular nose & max. when

follower is at apex of the nose.

→ Negetive sign ∗ shows that there is retardation when follower is in contact with nose of cam.

∗ Note :-

→ Tangent cam :-

∗ A symmetrical circular cam operating a flat-faced follower has following particulars :- Minimum radius of cam = 30mm, total lift = 20mm, Angle of lift = 75°, Nose radius = 5mm, speed = 600rpm Find acceleration of follower at beginning of lift, at end of contact with circular flank, at beginning of contact with nose & at apex of nose. Take radius of circular flank (R) = 82.4mm, $\phi = 34.2°$, OQ = 45mm.

→ $r_1 = 30mm$, total lift (JK) = 20mm, $\alpha = 75°$,

$r_2 = 5mm$, $N = 600rpm$, $\omega = \frac{2\pi \times 600}{60} = 62.84 rad/sec$,

$R = 82.4mm$, $\phi = 34.2°$, $OQ = 45mm$.

→ Acc. of follower at beginning of lift will come in (1) case

$\Rightarrow a_{max} = \omega^2 (R - r_1) = 206930 mm/sec^2$

$\therefore \quad a_{(max)} = 206.93 \text{ m/sec}^2$ ✓

→ Acc. of follower at end of contact with circular flank will come in (1) case.

$\Rightarrow a_{mini.} = \omega^2 (R - r_1) \cos\phi = 171130 \text{ mm/sec}^2$

$\therefore a_{mini} = 171.13 \text{ m/sec}^2$ ✓

→ Acc. of follower at beginning of contact with nose will come in (2) case.

$\Rightarrow a_{mini.} = -\omega^2 \times OQ \cos(\alpha - \phi)$

$= -134520 \text{ mm/sec}^2.$

$\therefore a_{mini.} = 134.52 \text{ m/sec}^2$ (Retardation) ✓

→ Acc. of follower at apex of nose will come in (2) case.

$\Rightarrow a_{max.} = -\omega^2 \times OQ = -177000 \text{ mm/sec}^2$

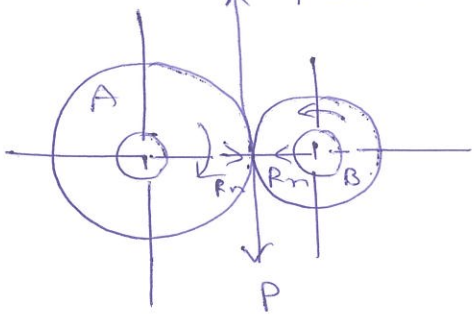
$\therefore a_{max.} = 177.7 \text{ m/sec}^2$ (Retardation) ✓

# Higher pair (Gears)

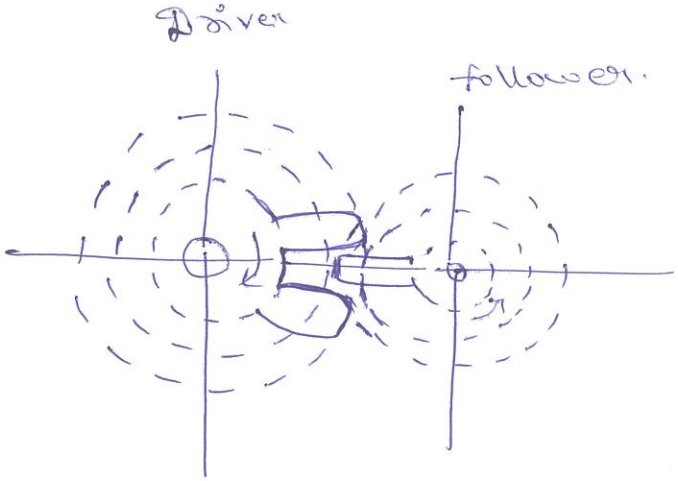
## * Introduction:-

→ Gears are nothing but toothed wheels.

→ A gear dive is provided when distance b/w diver & follower is very small.




Friction wheels




Gear / toothed wheels

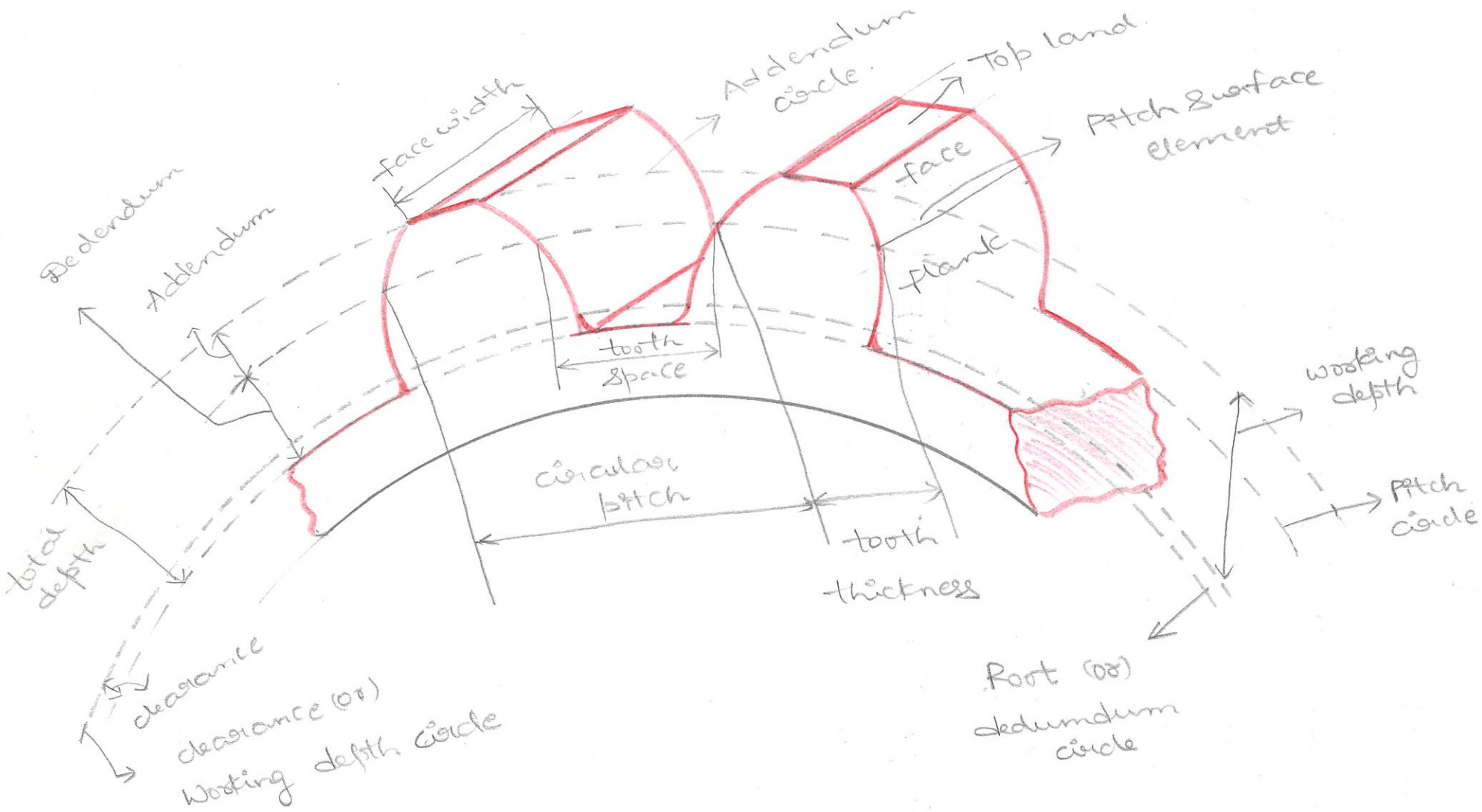
→ In friction wheels, wheel B is rotated by wheel A in opposite direction.

→ Wheel B is rotated as long as tangential force (P) exerted by wheel A does not exceed max. frictional resistance (F) b/w 2 wheels.

→ If P exceeds F, then slipping will take place.

→ In order to avoid slipping, teeth are provided.

→ So, friction wheel with teeth cut on it is known as gear / toothed wheel.




## Terms used in gears.

## * classification of toothed wheels :-

### 1). Spur gears :-

Two parallel shafts are connected by gears & gears have teeth parallel to axis of wheel.

### 2). Helical gears :-

Two parallel shafts are connected by gears & gears have teeth inclined to axis of wheel.

### 3). Bevel gears :-

Two non-parallel (or) intersecting are connected by gears.

### 4). Spiral gears :-

Two non-parallel (or) non-intersecting are connected by gears.

## * Terms used in gears :-

### 1). Pitch circle :-

It is an imaginary circle which by pure rolling action, would give same motion as the actual gear.

2). Pitch circle diameter :-

It is dia. of pitch circle. The size of gear is specified by pitch circle dia. It is also called as pitch diameter.

3). Pitch point :-

It is a common pt. of contact b/w two pitch circles.

4). Pressure angle / Angle of obliquity ($\phi$) :-

Angle b/w common normal to two gear teeth at pt. of contact & common tangent at pitch pt. Standard pressure angles are $14\frac{1}{2}^\circ$ & $20^\circ$.

5). Addendum :-

It is radial distance of a tooth from pitch circle to top of the tooth.

6). Dedendum :-

It is radial distance of a tooth from pitch circle to bottom of the tooth.

7). Addendum circle :-

circle drawn through top of teeth & is concentric with pitch circle.

8). Dedundum circle / Root circle :-

Circle drawn through bottom of teeth.

⇒ Root circle dia. = Pitch circle dia. × cos ϕ.

9). Circular pitch ($P_c$) :-

Distance measured on circumference of pitch circle from a pt. of one tooth to corresponding pt. on next tooth.

$$\Rightarrow P_c = \frac{\pi D}{T}$$

D → Dia. of pitch circle

T → No. of teeth on wheel

Note :- for two meshing gears,

$$P_c = \frac{\pi D_1}{T_1} = \frac{\pi D_2}{T_2}$$

$$\therefore \frac{D_1}{D_2} = \frac{T_1}{T_2}$$

10). Diametral pitch ($P_d$) :-

It is ratio of no. of teeth to pitch circle dia. in mm.

$$\therefore P_d = \frac{T}{D} = \frac{\pi}{P_c}$$

11). Module (m) :-

It is ratio of pitch circle diameter in mm to no. of teeth.

$$m = \frac{D}{T}$$

12). Clearance :-

It is radial distance from top of tooth to the bottom of tooth in a meshing gear. circle passing through top of meshing gear is known as clearance circle.

13). Total depth :-

It is radial distance b/w adderdum & dedendum circles of gear. Equal to Add. + Ded.

14). Working depth :-

It is radial distance from adderdum circle to clearance circle. Equal to sum of add. of two meshing gears.

15). Tooth thickness :-

It is width of tooth measured along pitch circle.

16). Tooth space :-

It is width of space b/w two adjacent teeth measured along pitch circle.

17). Face of tooth :-

It is surface of tooth above pitch surface.

18). Face width :-

It is width of gear tooth measured || to its axis.

19). Top land :-

It is surface of top of tooth.

20). flank of tooth :-

It is surface of teeth below the pitch surface.

* condition for constant velocity ratio of gears
→ law of gearing :-

→ In order to have a constant angular velocity ratio for all positions of wheels, P must be fixed pt. (pitch pt.) for 2 wheels.

→ condition is — common normal at pt. of contact b/w pair of teeth must always pass through pitch pt.

→ In designing the profiles for teeth of gear wheels, this condition must be satisfied.

→ This is also known as law of gearing.

$$\therefore \frac{\omega_1}{\omega_2} = \frac{D_2}{D_1} = \frac{T_2}{T_1}$$

## * Velocity of sliding of teeth :-

The velocity of sliding is the velocity of one tooth relative to its mating tooth along common tangent at point of contact.

$\therefore V_S = (\omega_1 + \omega_2)\, OP$

→ distance of point of contact from pitch pt.

## * Forms of Teeth :-

### 1). Cycloidal teeth :-

→ A 'cycloid' is curve traced by a point on the circumference of a circle which rolls without slipping on a fixed st. line.

→ When a circle rolls without slipping on outside of a fixed circle, curve traced by a pt. on circumference of a circle is known as 'epi-cycloid'.

→ If circle rolls without slipping on inside of a fixed circle, then curve traced by a pt. on the circumference of a circle is called 'hypo-cycloid'.

## 2). Involute teeth :-

→ An involute of a circle is a plane curve generated by a point on a tangent, which rolls on the circle without slipping. or

→ When power is being transmitted, max. tooth pressure (F) is exerted along the common normal through pitch pt.

→ This force (F) may be resolved into tangential & radial / Normal components.

→ These components act along at RE angles to common tangent to pitch circles.

⇒ Tangential force, $F_T = F\cos\phi$ → pressure angle.

⇒ Radial / Normal force, $F_R = F\sin\phi$

⇒ Torque exerted on gear shaft $= F_T \times r$ (r: pitch circle radius of gear.)

* A single reduction gear of 120 kW with a pinion 250 mm pitch circle dia. & speed 650 rpm is supported in bearings on either side. calculate total load due to power transmitted, pressure angle being 20°.

→ $P = 120\,kW = 120\times10^3\,W$, $d = 250\,mm$, $r = 0.125\,m$,

$N = 650\,rpm$, $\omega = \frac{2\pi\times650}{60} = 68\,rad/sec$, $\phi = 20°$.

$\Rightarrow P = T\omega \Rightarrow 120\times10^3 = T\times68$

$\therefore T = 1765\,N\text{-}m$ ✓

$\Rightarrow T = F_T \times r \Rightarrow F_T = \frac{T}{r} = 14120\,N$ ✓

$\Rightarrow F_T = F\cos\phi \Rightarrow F = \frac{F_T}{\cos\phi} = 15026\,N$ ✓

$\therefore F = 15026\,N$

* Note :-

→ circular pitch, $P_c = \pi m$. $\left(m = \frac{D}{T}\right)$.

→ Addendum circle radius of gear,

$R_A = R +$ Addendum.

→ Addendum circle radius of pinion,

$r_A = r +$ Addendum. ($r, R$ = pitch circle radius).

→ Angle turned through by gear/pinion

$= \frac{\text{length of arc of contact} \times 360°}{\text{circumference of gear/pinion}}$

→ Pitch circle radius of gear/pinion $(R/r) = \frac{mT}{2}$

$mt/2$.

## * Length of path of contact :-

→ Length of path of contact =
Length of path of approach +
Length of path of recess.

→ Length = $\sqrt{R_A^2 - R^2\cos^2\phi} + \sqrt{r_A^2 - r^2\cos^2\phi} - (R+r)\sin\phi$.

(∵ Approach length = $\sqrt{R_A^2 - R^2\cos^2\phi} - R\sin\phi$
& recess length = $\sqrt{r_A^2 - r^2\cos^2\phi} - r\sin\phi$).

## * Length of arc of contact :-

→ Length of arc of contact =
Length of path of contact / $\cos\phi$.

## * Contact ratio (or no. of pairs of teeth in contact) :-

→ It is defined as ratio of length of arc of contact to the circular pitch.

→ Contact ratio = $\frac{\text{Length of ~~path~~ arc of contact}}{P_c}$

($\because P_c = \pi m$, m → module)

* The no. of teeth on each of two equal spur gears in mesh are 40. The teeth have 20° involute profile & module is 6mm. If arc of contact is 1.75 times the circular pitch, find the addendum.

→ $T = t = 40$, $\phi = 20°$, $m = 6\,mm$

$\Rightarrow P_C = \pi m = \pi \times 6 = 18.85\,mm$

Given $\Rightarrow$ length of arc of contact $= 1.75 \times P_C$

$= 1.75 \times 18.85 = 33\,mm$

$\Rightarrow$ length of path of contact = length of arc of contact $\times \cos\phi$ $= 33 \cos 20° = 31\,mm$

$\Rightarrow (R = \gamma) = \left(\frac{mT}{2} = \frac{mt}{2}\right) = \frac{6 \times 40}{2} = 120\,mm$

$\Rightarrow$ length of path of contact $= 31 =$

$$\sqrt{R_A^2 - R^2\cos^2\phi} + \sqrt{\gamma_A^2 - \gamma^2\cos^2\phi} - (R+\gamma)\sin\phi$$

$$\Rightarrow 31 = 2\left[\sqrt{R_A^2 - R^2\cos^2\phi} - R\sin\phi\right] \quad (\because R = \gamma, R_A = \gamma_A)$$

$$\Rightarrow \frac{31}{2} = \sqrt{R_A^2 - (120)^2 \cos^2 20°} - 120\sin 20°$$

$$\therefore R_A = 126.12\,mm$$

$\therefore$ Addendum $= R_A - R = 126.12 - 120 = 6.12\,mm$

* A pinion having 30 teeth drives a gear having 80-teeth. The profile of gears is involute with 20° pressure angle, 12mm module & 10mm addendum. Find length of path of contact, arc of contact, contact ratio, angle turned through by gear & pinion, max. velocity of sliding (pitch line speed (v) = 1.2m/sec)

→ $t = 30$, $T = 80$, $\phi = 20°$, $m = 12$ mm, Addendum = 10mm

$v = 1.2$ m/sec.

Length of path of contact :-

$$\Rightarrow r = \frac{mt}{2} = \frac{12 \times 30}{2} = 180 \text{ mm}$$

$$\Rightarrow R = \frac{mT}{2} = \frac{12 \times 80}{2} = 480 \text{ mm}$$

$$\Rightarrow r_A = r + \text{Addendum} = 180 + 10 = 190 \text{ mm}$$

$$\Rightarrow R_A = R + \text{Addendum} = 480 + 10 = 490 \text{ mm}$$

$$\text{length of path of contact} = \sqrt{R_A^2 - R^2 \cos^2\phi} + \sqrt{r_A^2 - r^2 \cos^2\phi} - (R + r)\sin\phi.$$

$$\therefore \text{length} = 52.3 \text{ mm} \checkmark$$

length of arc of contact :-

= length of path of contact / $\cos\phi$

$= \frac{52.3}{\cos 20^\circ} = 55.66\text{mm}$ ✓

contact ratio :-

$\Rightarrow P_c = \pi m = \pi \times 12 = 37.7\text{mm}$

$\therefore$ contact ratio $= \frac{\text{length of arc of contact}}{P_c}$

$= \frac{55.66}{37.7} = 1.5 \simeq 2$ ✓

Angle turned :-

⇒ Angle turned through by gear

$= \frac{\text{length of arc of contact} \times 360^\circ}{\text{circumference of gear}}$

$= \frac{55.66 \times 360^\circ}{2\pi 480} = 6.647^\circ$ ✓

⇒ Angle turned through by pinion

$= \frac{55.66 \times 360^\circ}{2\pi 180} = 17.726^\circ$ ✓

## Max. velocity of sliding :-

⇒ We know, $V = \omega_1 r = \omega_2 R$

⇒ $\omega_1 = \frac{V}{r} = \frac{1.2}{0.18} = 6.66$ rad/sec.

⇒ $\omega_2 = \frac{V}{R} = \frac{1.2}{0.48} = 2.5$ rad/sec.

$\therefore V_S = (\omega_1 + \omega_2)$ length of path of approach

(∵ Approach > Recess).

$= (6.66 + 2.5)\, 27.3 = \overset{250}{\cancel{455.4}}$ mm/sec.

$\therefore V_S = \overset{250}{\cancel{455.4}}$ mm/sec ✓

## * Interference in involute gears :-

→ The phenomenon when the tip of tooth undercuts the root on its mating gear is known as interference.

→ Interference may only be prevented, if addendum circles of the two mating gears cut the common tangent to the base circles b/w the pts of tangency.

→ In such a case, max. length of path of contact $= (r+R)\sin\phi$ & max. length of arc of contact $= (r+R)\tan\phi$.

* Minimum no. of teeth on the pinion in order to avoid interference :- (t)

→ let $A_p m$ = Addendum of pinion, where $A_p$ is a fraction by which standard addendum of one module for pinion should be multiplied in order to avoid interference.

→ $$A_p m = \frac{mt}{2}\left[\sqrt{1+\frac{T}{t}\left(\frac{T}{t}+2\right)\sin^2\phi}-1\right]$$

→ $$t = \frac{2A_p}{\sqrt{1+a(a+2)\sin^2\phi}-1}$$

where, $a = \frac{T}{t} = \frac{R}{r}$, $r = \frac{mt}{2}$

→ If pinion & wheel have equal teeth, then $a=1$, then $t = \frac{2A_p}{\sqrt{1+3\sin^2\phi}-1}$

* Minimum no. of teeth on the wheel in order to avoid interference :- (T)

→ let $A_w m$ = Addendum of wheel.

→ $$A_w m = \frac{mT}{2}\left[\sqrt{1+\frac{t}{T}\left(\frac{t}{T}+2\right)\sin^2\phi}-1\right]$$

Where, $\phi$ = pressure angle / Angle of obliquity

$$\rightarrow T = \frac{2\,Aw}{\sqrt{1+\frac{1}{a}\left(\frac{1}{a}+2\right)\sin^2\phi}-1}$$

→ If wheel & pinion have equal teeth, then $a=1$, then $T = \frac{2Aw}{\sqrt{1+3\sin^2\phi}-1}$

* Determine the minimum no. of teeth required on a pinion, in order to avoid interference which is to gear with,

1). a wheel to give a gear ratio of 3 to 1 &

2). An equal wheel.

The pressure angle is 20° & a standard addendum of 1module for wheel may be assumed.

→ $a = \frac{T}{t} = 3$, $\phi = 20°$, $Aw = 1module$.

1). $a = \frac{T}{t} \Rightarrow t = \frac{T}{a} = \frac{2 \times Aw}{a\left(\sqrt{1+\frac{1}{a}\left(\frac{1}{a}+2\right)\sin^2\phi}-1\right)}$

$$\Rightarrow t = \frac{2\times 1}{3\left(\sqrt{1+\frac{1}{3}\left(\frac{1}{3}+2\right)\sin^2 20°}-1\right)} = 15.04$$

$\therefore t = 16$ ✓

2) $G = \frac{T}{t} \Rightarrow t = \frac{T}{G} = \frac{2 \times Aw}{G\left(\sqrt{1+3\sin^2\phi} - 1\right)}$

$\Rightarrow t = 4.33$

$\therefore t = 5$ ✓

→ * A pair of spur gears with involute teeth is to give a gear ratio of 4:1. The arc of approach is not to be less than the circular pitch & smaller wheel is the driver. The angle of pressure is 14.5°. Find 1). least no. of teeth that can be used on each wheel & 2). addendum of wheel in terms of circular pitch & pinion also.

→ $G = \frac{T}{t} = \frac{R}{r} = 4, \phi = 14.5°$

1). Max. length of arc of approach $= r \tan\phi$.

& $P_c = \pi m = \frac{\pi 2r}{t}$ $\left(\because r = \frac{mt}{2} \Rightarrow m = \frac{2r}{t}\right)$.

Given as, $r \tan\phi = \frac{2\pi r}{t}$

$\Rightarrow t = \frac{2\pi}{\tan\phi} = \frac{2\pi}{\tan 14.5°} = 24.3 \simeq 25$ ✓

$\therefore T = Gt = 4 \times 25 = 100$ ✓

2). $A_{wm} = \frac{mT}{2}\left[\sqrt{1+\frac{t}{T}\left(\frac{t}{T}+2\right)\sin^2\phi} - 1\right]$

$= \frac{m\times 100}{2}\left[\sqrt{1+\frac{25}{100}\left(\frac{25}{100}+2\right)\sin^2 14.5^\circ} - 1\right]$

$= 0.85m = 0.85 \times \frac{P_c}{\pi} = 0.27 P_c$

$\therefore A_{wm} = 0.27 P_c$ ✓

& $A_{pm} = \frac{mt}{2}\left[\sqrt{1+\frac{T}{t}\left(\frac{T}{t}+2\right)\sin^2\phi} - 1\right]$

$= \frac{m\times 25}{2}\left[\sqrt{1+\frac{100}{25}\left(\frac{100}{25}+2\right)\sin^2 14.5^\circ} - 1\right]$

$=$

$\therefore A_{pm} =$

* Two gear wheels mesh esternally & are to give a velocity ratio of 3 to 1. The teeth are of involute form, module = 6mm, Addendum = one module, pressure angle = 20°. The pinion rotates at 90 rpm. Determine :-

1). No. of teeth on the pinion to avoid interference on it & corresponding no. of

teeth on the wheel 2). Angular speed of wheel.

→ $a = \frac{T}{t} = 3$, $m = 6mm$, $Ap = Aw = 1 module = 6mm$,

$\phi = 20°$, $N_1 = 900 rpm$

$\Rightarrow \omega_1 = \frac{2\pi 90}{60} = 9.43 rad/sec.$

1). $$t = \frac{2Ap}{\sqrt{1 + a(a+2)\sin^2\phi} - 1}$$

$\therefore t = 18.2 \simeq 19$ ✓

$\Rightarrow T = at = 3 \times 19 = 57$ ✓

2). $$\frac{\omega_1}{\omega_2} = \frac{T}{t}$$

$\Rightarrow \omega_2 = \omega_1 \times \frac{t}{T} = \frac{9.43 \times 19}{57}$

$\therefore \omega_2 = 3.14 rad/sec.$ ✓

* Minimum no. of teeth on a pinion for involute rack in order to avoid interference :-

→ Rack is a part of toothed wheel of infinite dia.

→ So, its base circle dia. & profiles of involute teeth are straight lines.

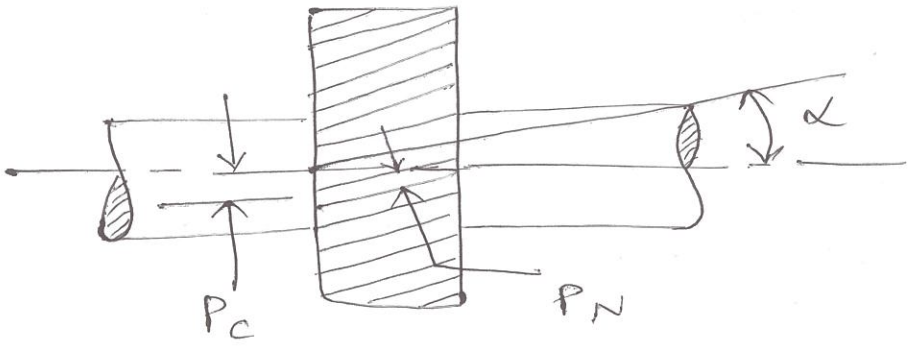
→ Addendum for rack, $A_{pm} = \frac{mt}{2}\sin^2\phi$.

→ $t = \frac{2AR}{\sin^2\phi}$.

## * Helical Gears :-

→ A helical gear has teeth in the form of helix around the gear.

→ Two such gears may be used to connect two parallel shafts in place of spur gear.




→ Normal pitch ($P_N$) :-

It is distance b/w similar faces of adjacent teeth, along a helix on pitch cylinder normal to the teeth.

→ Axial pitch ($P_C$) :-

It is distance measured parallel to the axis, b/w similar faces of adjacent teeth. It is same as circular pitch & it if 'α'

is helix angle, then circular pitch ($P_C$) is -

$$\therefore P_C = \frac{P_N}{\cos\alpha}$$

$\alpha \rightarrow$ helix angle / spiral angle.

## * Bevel gears :-

→ Bevel gears are used for transmitting power at a constant velocity ratio b/w two shafts whose axes intersect at a certain angle.

## * Worm gears :-

→ Worm gears are used for transmitting power at high velocity ratio's b/w non-intersecting shafts.

→ It is mostly used as a speed reducer, which consists of worm & a worm wheel (or) gear.

→ Worm is usually of a cylindrical form having threads of same shape as that of an involute rack.

→ Worm wheel (gear) is similar to helical gear with a face curved to conform to the shape of the worm.